

What is Algebra

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Abstract: The manuscript defines algebra as the language of higher mathematics, with the rules of logic imbedded. We discuss why the textbooks on algebra routinely fail to define the subject. The paper shows that algebra is a tool for reasoning in that it enables the connection of one statement to another. We also demonstrate that the language of algebra is an elegant tool for conveying information in the most precise and concise manner possible

If calling things by their right name is the beginning knowledge, then there is a crucial reason that the first thing the students learn is a definition of the subject of algebra. It is a mystery why anyone would write a textbook on algebra and routinely fail to define the subject. So what is algebra? Algebra is the language of higher mathematics, but it is not just another language. Algebra is an abstract symbolic language plus reasoning; it is a language with the rules logic built-in. The structure of algebra, assumption-deduction-conclusion, is based on the concept of formal proof, a human invention that dates from the fourth century B.C.E. An equation in algebra is equivalent to a sentence in a written language that has a quantitative connotation; such sentences can be translated into equations. For example, the statement: John is five years older than Mary, translates into: $J = 5 + M$, where the literals, J and M, are *John* and *Mary's* ages, the verb “is” translates to an equals, = sign, and the words “*five years older*” translates to 5 and the addition operator, 5 +. Algebra is a tool for reasoning in that it enables the connection of one statement to another. The additional information needed to proceed is the statement: two years ago, John was twice as old as Mary. Both statements can now be connected through a process of reformulation and the unknown ages can be deduced from the one-to-one matching of terms.

First statement: $J = 5 + M \rightarrow J - 2 = 5 + M - 2 + 2 - 2 \rightarrow J - 2 = 7 + M - 4$

Second statement: $J - 2 = 2(M - 2) \rightarrow J - 2 = M - 2 + M - 2 \rightarrow J - 2 = M + M - 4$

The visual method above does not solve for the variable or use any abstract symbolic logic but reformulates and compares the equations to deduce that Mary's age is 7, and John is $5 + 7$ or 12. Two years ago, John was 10 and Mary was 5, that is, John was twice as old as Mary.

The language analogy requires the following definitions: the set of real numbers is the nouns of the language. The set is infinite and unique, with a powerful base 10 numerations that enables every number in the set to be expressed as a decimal fraction. The concept of infinity is a human invention that does not exist; consequently, to enable student visualization, the sand on a beach is a reasonable facsimile of the "infinite" set of real numbers, where each grain of sand has a unique number inscribed. The set contains two numbers, zero and one, that have special properties that enable the process of reformulation. Everyone thinks they know what the numbers are until it comes time to define them. When the students are asked to define the number 3 without using the word three in their definition, there is no response. A student will occasionally attempt to define the number 3 as $1 + 1 + 1$, but then he or she is required to define the number 1 and the operation of addition. The words used to define 1 and the operator $+$ must in turn be defined, so the student would soon enter an endless loop of definitions. The difficult, if not impossible, task of trying to define the numbers can be avoided with the hypothesis that the numbers, whatever they are, satisfy certain basic assumptions. In the first textbook on mathematics, the *Elements*, by Euclid, and used at the University of Alexandria in 310 B.C.E., 5 assumptions are postulated to develop the early geometric algebra: *Things that are equal to the same thing or to equal things are equal to each other. If equals are added to equals, then the results are equal. If equals are subtracted from equals, then the remainders are equal. Things that coincide with one another are equal to one another. The whole is greater than the part.* These assumptions are the foundation of algebra and are therefore, the grammar and syntax of the language. The operators, $+$, $-$, $*$, \div , $=$, $\sqrt{\quad}$, and so on, tell us what action to take, so they play the role of verbs in the language.

All mathematical systems are a human invention; algebra deals essentially with quantities associated with the physical reality: matter, energy, space and time. Some examples are: 2 people (matter-energy, if alive, see **Note***), 5 pounds of potatoes (matter-potential energy), 55 miles per hour (space-time), 3

100-watt light bulbs (matter-energy), and 40 inches long (space). Like all languages, algebra is composed of concepts that attempt to capture the essence of reality. A concept is the basic element of thought, consequently, mathematical concepts such as the Equator, the North Pole, the number five, or any number do not exist in the physical reality. The most useful application of algebra is in the fundamental laws of nature; there are about a dozen such laws: (3) Motion, (4) Electromagnetism, (1) Gravity and so on. These laws are universal and must be expressed in abstract algebraic form so that the field of applicability is entirely unrestricted. In *The Mathematical Principles of Natural Philosophy* (nature's philosophy is what physics was called in those days), by Isaac Newton, published by Cambridge University in 1687, the second law of motion is expressed in the form of analytical description and devoid of any mathematical formulation:

The alteration of the quantity of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.

The same law in algebraic form: $f = \frac{d}{dt}(mv)$, the physical interaction is:

$$force = \frac{matter \times space}{(time)^2}$$

Where velocity v in the above equation is the quantity of space traversed per unit time. Physics is analytical description, it is not mathematics; however, the language of algebra is an elegant tool for conveying information; it is not only more precise and concise than any analytical description can be but also shows the relationship between the components of the physical reality. Newton's 2nd law of motion states that the impressed force (f) is (=) ever proportional to the time rate of change (d/dt) of the linear momentum (mv). Newton's definition of momentum: "*The quantity of motion is the measure of the same, arising from the velocity and quantity of matter conjunctly*", hence the analytical description of the law and the algebraic equation have exactly the same meaning. Newton knew how to differentiate (d/dt) having invented fluxions (calculus) before his 25th birthday. Newton's book was devoid of his mathematical invention because he wanted people to understand what he wrote. Historically, this was the first

book on physics that was correct; it was the catalyst for the Industrial Revolution.

Note:* In addition to matter-energy, perhaps spirituality. What the experiments in quantum physics reveal is that the most basic unit of matter is a closely bound nucleus of protons and neutrons coupled to electrons that are at enormous distances from the nucleus compared to the particle sizes. If any human were compressed indefinitely in three directions such that every electron is pushed up against its nucleus and every nucleus is pushed up against its neighbor, he or she would not be visible. We humans have no substantial existence physically, our existence is 100% spiritual by default; I do not have the vaguest idea what that means so no attempt will be made to define spirituality. In apparent reality, the reality that we see, what we perceive to be solid matter is largely empty space. For a explanation in layman's terms of reality at the micro-level, two books are suggested: *QED (Quantum Electro-Dynamics)*, *The Strange Theory of Light and Matter*, Princeton University Press and *The Character of Physical Law*, Massachusetts Institute of Technology Press, by Richard P. Feynman, Professor of Physics and Nobel Laureate, 1918-1988. Feynman writes in *The Character of Physical Law*, "On the other hand, I think I can safely say that nobody understands quantum mechanics", page 129. Reality at the micro-level is strange and remote from our experience; it is beyond all human conceptual elaboration or understanding.

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