

**INCORRECT STEPS, CORRECT ANSWER**

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Algebra teachers always tell their students that only factors of the entire numerator and the entire denominator may be “cancelled” in a fraction. Terms in the numerator and denominator or factors of individual terms may not be safely cancelled. For instance suppose that an algebra class is asked to rename the fraction:

$$\frac{2+2x}{4+4x}$$

Most teachers would expect the simplification to proceed as follows:

$$\frac{\cancel{2}(1+x)}{\cancel{4}(1+x)} = \frac{\cancel{2}}{\cancel{4}} = \frac{1}{2}$$

But there are four other “simplifications” which teachers are unlikely to accept, but which students might try:

$$1. \frac{2+2x}{4+4x} = \frac{{}^1\cancel{2}+2x}{{}_2\cancel{4}+4x} = \frac{1+2x}{2+4x}$$

$$2. \frac{2+2x}{4+4x} = \frac{2+2^1x}{4+4_2x} = \frac{2+x}{4+2x}$$

$$3. \frac{2+2\cancel{x}}{4+4\cancel{x}} = \frac{2+2}{4+4} = \frac{4}{8}$$

$$4. \frac{2+2x}{4+4x} = \frac{2+2^1\cancel{x}}{4+4_2\cancel{x}} = \frac{2+1}{4+2} = \frac{3}{6}$$

In each case, terms or factors of terms divide out, contrary to the normal procedures of algebra. Amazingly, however, each yields the correct answer.

Methods 3 and 4 obviously simplify to the correct answer,  $\frac{1}{2}$ . Methods 1 and 2 also yield results which can be correctly simplified further to yield the correct final answer. For instance, for (2):

$$\frac{2+x}{4+2x} = \frac{(2+x)}{2(2+x)} = \frac{1}{2}$$

The moral of this story is to be very careful when choosing examples to show that certain procedures are incorrect.

Challenge to the reader: Look for other incorrect procedures which yield correct results in special cases.

David Duncan and Bonnie Litwiller recently retired as Professors of Mathematics at the University of Northern Iowa. During their long careers they taught numerous courses for pre-service and in-service mathematics teachers and held several offices in the Iowa Council of Teachers of Mathematics. Dr. Litwiller also served as a Director of the NCTM.