WHO WANTS TO BE A MILLIONAIRE: ANALYZING THE EFFECTS OF SAVINGS PATTERNS

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Abstract: This article examines ways in which yearly deposits could yield a total amount of $1,000,000. Various possibilities of the deposit amounts, the number of deposits, and the interest rate earned by these deposits are considered in this analysis.

A financial commentator recently commented on the radio that a person could accumulate a million dollars by simply saving a thousand dollars a year for forty years. This assertion gives rise to an intriguing mathematical question: what interest rate would be necessary in order to accomplish this goal?

Let us suppose that $1,000 is deposited in a savings account at the beginning of each of 40 years. Suppose further that this account pays an interest rate of R (expressed decimally) compounded each year. At the end of 40 years, the original deposit of $1,000 would amount to $1000(1+R)^{40}$ dollars. The second deposit would earn interest for 39 years and would amount to $1000(1+R)^{39}$ at the of the 40-year period. The same principle (with decreasing exponents) would apply to each of the subsequent deposits; the 40th deposit would yield $1000(1+R)^{1}$ at end of the that 40th year.

Summarizing:

Year	Value of Deposit in That Year at the End of 40 Years
1	$1000\,(1+R)^{40}$
2	$1000(1+R)^{39}$
3	$1000(1+R)^{38}$
.	.
.	.
.	.
39	$1000(1+R)^{2}$
40	$1000(1+R)^{1}$

In total, these 40 deposits would amount to:

T (Total)= $1000(1+R)^{40} + 1000(1+R)^{39} + \ldots + 1000(1+R)^{1}$ at the end of the fortieth year.

To evaluate T, we first multiply both sides by (1+R), this yielding:

$$(1+R)T=1000(1+R)^{41} + 1000(1+R)^{40} + \ldots +1000(1+R)^{2}$$

Subtracting these progressions yields:

$$(1+R)T-T=1000(1+R)^{41}-1000(1+R)^{1}$$

Rewriting: $T+RT-T=1000[(1+R)^{41}-(1+R)]$

(Formula 1) $$T = \frac{1000(1+R)[(1+R)^{40}-1]}{R}$$

Let us first evaluate T in Formula 1 for the currently common interest rate of R=5% or 0.05. For this assumption,

(Formula 2) $$T = \frac{1000(1.05)[(1.05)^{40} - 1]}{0.05}$$

$$= \$126,839.76$$

This is well short of the desired million dollars. Using a much more optimistic assumption of R=10%= 0.10, we achieve

$$T = \frac{1000(1.10)[(1.10)^{40} - 1]}{0.10}$$

$$= \$486,851.81$$ still far short of our goal.

Repeated calculations using increasing values of R reveal that T will first exceed $1,000,000 for R= 12.53% (to 2 decimal places). For this exceptional interest rate T = $1,000,388.25.

This calculation casts serious doubt on the commentator's prediction as a reasonable plan; to achieve a 12.53% potential interest rate an exceedingly risky investment would be needed.

Let us reconsider this problem by fixing the interest rate at the more reasonable 5% and altering the number of years or the periodic deposit.

First, assume $1000 yearly deposits and a 5% interest rate. Consecutive calculations in Formula 2 for increasing exponents reveal that a total of $1,000,000 is first attained after 80 years (T=$1,019,790.26). This would surely exceed the normal savings time-span for most people.

In tabular form:

Year	Value of Total Deposits (each $1000) Earning 5% annual interest
40	$126,839.76
41	$134,231.75
42	$141,993.34
.	.
.	.
.	.
78	$923,027.45
79	$970,228.82
80	$1,019,790.26

Secondly, let us find what yearly deposits would suffice for a 40-year time-span and a 5% interest rate. Adjusting Formula 2 by replacing 1000 with X and T with 1,000,000 yields:

$$1{,}000{,}000 = \frac{X(1.05)[(1.05)^{40}-1]}{0.05}$$

$$X = \frac{(1{,}000{,}000)(0.05)}{(1.05)[(1.05)^{40}-1]} = \$7{,}883.96$$

In summary, a million dollar savings account is possible, but it would require either a much higher interest rate, a much longer time span, or a much larger deposit than that suggested by the commentator.

The reader and his/her students are encouraged to investigate other questions involving compound interest.

David Duncan and Bonnie Litwiller recently retired as Professors of Mathematics at the University of Northern Iowa. During their long careers they taught numerous courses for pre-service and in-service mathematics teachers and held several offices in the Iowa Council of Teachers of Mathematics. Dr. Litwiller also served as a Director of the NCTM.