

To “Commonize” or Not to “Commonize” That is the Question

Lou Schultz
Southeastern Louisiana University

Abstract: Research is a journey of discovery. This article follows a fourth and a fifth grade teacher through the unique experience of conducting mathematics related research during their summer break and then using their new understanding to create a meaningful lesson. The journey starts with their selection of an appropriate research question then wanders through the agony of research and the need to find the answers to even more questions in order to answer the original question. The teachers experience the joy of discovery and gain valuable pedagogical insight at the same time. The final step was to use their new knowledge and insight to create a dynamic lesson and test then it on junior high students. Both participants felt that they had come away from this experience a better teacher.

Selecting a Research Question

We are working with a fourth and a fifth grade teacher who realized that there were many mathematical concepts worthy of further study. The teachers also knew that the upper elementary grades were incubators of mathematical knowledge and that the success of bringing this knowledge to life within their students was dependent on their own conceptual understanding. What one mathematical concept should they spend this valuable opportunity exploring?

After spending a day discussing and debating various possibilities, there seemed to be an almost natural funneling toward fractions. Using a logical process that is typical of good teachers, they discussed what would help them better meet the needs of their students when it came to fractions. During the discussions both teachers agreed that many students have a hard time understanding why we only use common denominators in certain types of problems. From these initial discussions the teachers narrowed their option to one question: “Why do we need common denominators to add and subtract fractions but not multiply or divide them?”

Researching the Question

The teachers had determined the question to investigate and were allowed to pursue this investigation on their own. They would meet daily with their faculty advisor to discuss progress, get clarification - if needed - on articles they had read, brainstorm ideas, and discuss their next steps. After each morning meeting, they would either work together in the mathematics department or go to the library where they would research and discuss topic related to their own investigation. The role of the faculty member during this time was to provide support in the form of encouragement, concept clarification, and help in understanding the role prior knowledge plays in conceptual understanding. All of the research was accomplished by the teachers with the customary help provided by the Southeastern University library staff. During the morning discussions the faculty member used questioning techniques to help deepen the teacher's understanding of the concepts being investigating. One teacher commented after the project that she now used the same questioning technique with her own students and they were becoming better thinkers and less reliant on her for the answers.

After the teachers had selected fractions as their area of research, it was decided that they needed a deeper understanding of the following questions: What is the meaning of addition, subtraction, multiplication, and division? Is there a common pattern when we perform these operation using positive integers or fractions? If so what is the Pattern? What would happen if we started with any integer and repetitively perform any of these operations using (1) a natural number and (2) a proper fraction? They soon discovered that using tables to look at the patterns was not always the best method.. Graphs were soon incorporated into their research. From the use of graphs the concepts of convergent

and divergent came to light. These concepts were new to experienced teachers whose formal training offered limited college level mathematical training. They found the concepts easy to understand but had never been exposed to them. The teachers remarked how a simple thing such as using both tabular and graphical representations changed the understanding level of the learner. The project was not only providing new conceptual understanding but was also providing pedagogical insight.

The teachers felt they had the background knowledge needed to focus on their research question. The most important thing that happened at this time was the teacher discussions where the teachers now using new knowledge to justify their statements. They were excited, as was I, with a higher level of thinking about something that we take for granted. Conceptual understanding was very evident. One of the teachers said “We should talk more about common units not just common denominators. It would help our students make a connection to what they have already learned.” Operations with fractions had leaped from the skills column to the concept column. These are good teachers but for many years we produced elementary teachers proficient in teaching skills not concepts. This has changed drastically since the 1989 introduction of the NCTM Standards

Analysis of Research

What they had rediscovered in their research was that the four basic mathematical operations are all based on addition. Therefore, what applies to addition must also apply to the other operations. Their research showed that you can add apples and oranges but first you must change them to the a common unit called fruit. $\frac{5}{8}$ means you have 5 $\frac{1}{8}$ ths thus $\frac{1}{8}$ is the unit and by the rules of addition it must be added to a value given in terms of the number of $\frac{1}{8}$ ths or they must both be changed to another common . Since

the value of the unitary fraction depends on the denominator we say we must have a common denominator. They realized that since multiplication is repeated addition of the same value you already have a common denominator. They also realized that division is multiplying by a reciprocal therefore the same logic applies to it as when multiplying by a fraction. Their general conclusion was that there is no difference between adding, subtracting, multiplying, or dividing by a fraction. They all involve a common unit.

Figure 1 is a visualization of the path the teachers followed in their quest of discovery.

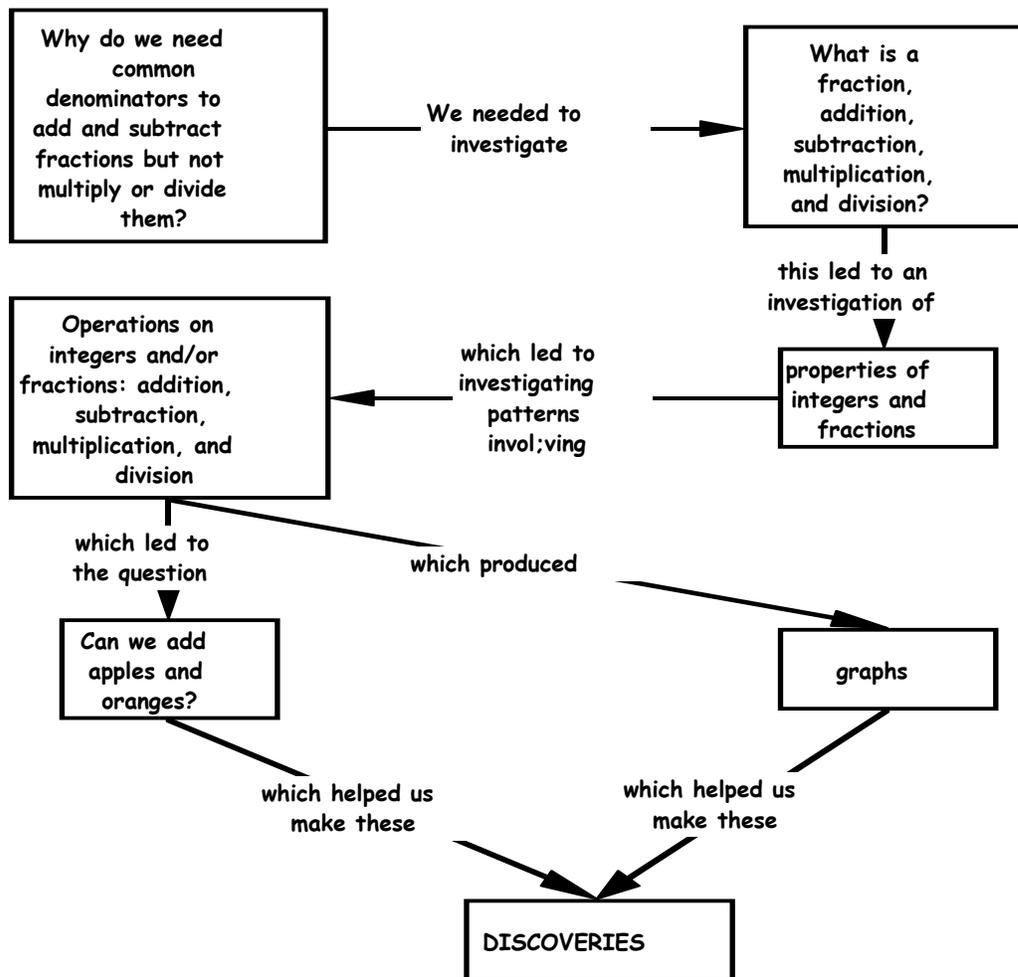


Figure 1

As you can easily see, the path to finding the answer to the research question was not direct. A clear understanding of what we often take for granted was needed if they were to reach their goal. As teachers they had an understanding of the importance of prior knowledge but this project helped reemphasize the importance of the little concepts that must be understood before a major concept takes shape. It also reinforced the idea of “teachable moments” when questions asked lead to discovery and discussion of other concepts that came about as a tangent to what was being originally discussed.

Presentation of Findings

After almost two weeks of research and analysis of information, the two teachers presented their finding to the entire project staff and other participants. They started with a review of how they decided on their research question followed by an explanation of the background material they needed in order to complete their investigation. Their presentation ended with a restatement of their research question followed by an explanation of what their research had shown. As with all good research, the researchers then opened the floor for a critique of their procedures and findings.

Incorporating New Knowledge into a Lesson

The next phase was for the teachers to take this new knowledge and use it to improve their teaching. The teachers were given two weeks to prepare lessons that would effectively incorporate their research into a classroom lesson. Although they were developing lessons that would be used with their fourth and fifth graders, they were told that their lesson would be piloted by seventh graders due to student availability. They now feared that the lessons would be too easy for seventh grade students and that it would be impossible to properly critique the lesson’s impact on learning. In addition to

incorporating their new knowledge into an appropriate lesson the teachers also had to insure that the lessons would be tied to the Louisiana Benchmarks.

They decided to begin their lesson with a five minute exercise that would assess the students’ prior knowledge of the concepts of addition and subtraction. The teachers would then discussed with the students the meaning of addition and subtraction developing the concepts properly and clarifying any misconceptions. During this discussion they would also probe student understanding of the use of units in addition and subtraction. Several apples and oranges would be placed on a table and the students asked to add them together and give an appropriate unit, this would be repeated with several sets of different objects in anticipation that the students would quickly realized they could always give a sum if they could express it in terms of a common unit. This same procedure would be repeated for subtraction. The students would next be asked to develop a rule for addition and subtraction based on what they had just learned. The rule would not be given to the students but rather developed by the students giving them ownership.

The use of fraction strips would now be introduced to enhance the students visualization of addition and subtraction of fractions using common units. The use of containers marked in units of $\frac{1}{6}^{\text{th}}$ would be used in conjunction with the fraction strips to help the students visualize the addition and subtraction of like unit fractions.

The lesson will end with a discussion of the concepts of addition and subtraction using common units. The students will be asked to restate the rules that they developed and how these rules apply to various sets of numbers. They will also be asked to discuss

when we can not add or subtract to insure complete understanding of the concepts covered.

Testing the Lesson with Students

Even though the lesson was developed for students at a lower grade level, the students who participated in the project said that they still had problems with fractions. I must say here that these were not low performing students but students who were working at or above grade level. The students were intrigued with the idea that they were adding and subtracting fractions based on a common unit. They stated that when they approached these operations from this aspect that it made more sense and was easier to understand why we needed a common denominator. It was no longer just something that we “had to do.” They were also able to see the connection between the addition and subtraction of fractions and that of integers. Due to the maturity of these students the investigation and concepts was expanded to include other numerical representation such as decimals, mixed numbers, and improper fractions. The teachers were pleasantly surprised when the students made other connections based on their new knowledge. Even though the material was originally developed for younger children it provided an opportunity for older children to fill in the gaps in their conceptual understanding. The skills were present but the understanding needed to be mentally “mathematically comfortable” were lacking.

Critiquing the Project

This project has surpassed our expectations. It provided an opportunity for teachers to explore mathematical concepts in a new and rewarding manner. This project has shown that research is not only a means of adding to the existing knowledge pool but

is also a very effective teaching tool. In depth knowledge provides a master that instills confidence in the instructor. The teachers working with fractions commented on their growth as teachers and the fact that their students would benefit from their new insight. They believed that their research would enable them to add conceptual understanding to mathematical operations involving fractions.

References

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Dr. Lou Schultz received his Ph. D. in Curriculum and Instruction from Louisiana State University. He is a faculty member of the Department of Mathematics, Southeastern Louisiana University. Dr. Schultz is a much sought-after consultant to K-12 school systems in Louisiana.