Assessment of Students' Learning in Plane & Solid Geometry

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Abstract

The diversity of learning styles and strengths of students at all grade levels (Gardner, 1983) creates a natural environment where individual students will differ in the ways they are able to exhibit what they have learned in mathematics. There is a variety of assessment techniques that teachers can use when gathering information and to make inferences and judgments about students' learning and to also use this information for grading purposes. This article describes six assessment techniques that teachers of mathematics can use to ascertain information about students' strengths and weaknesses in geometry, to make necessary changes in instruction and curriculum to improve students' geometry learning, and to assign grades to students' work.

Introduction

Facilitating planned and spontaneous assessment is the means by which teachers gather information about what and how students are learning. Authentic (true, genuine, real) information about students' learning is a vital companion to and necessary part of curriculum and instruction. Assessment results provide information that teachers can use to make changes, adjustments, and improvements in curriculum and instruction to meet the needs of individual students as well as the whole class and to document students' progress and achievement through grading. For example, assessment of students' abilities to categorize plane figures (e.g., triangle, quadrilaterals, parallelograms, pentagons, etc.) might lead to a decision to (1) re-teach various concepts (e.g., polygons, sides, angles, parallelism), (2) develop exploration centers to review and reinforce the topic, (3) provide more hands-on experiences for the next geometry topic as a preventive measure, and/or (4) make adjustments in the learning tools used to accommodate special needs of students (e.g., manipulatives (larger if needed) for students with fine motor skill difficulties).

From the learners' point of view, assessment is the way they are able to convey their understandings and misunderstandings to the teacher. Students appreciate opportunities to show what they have learned when they are able to do so in a caring, nurturing and safe environment. They are more accepting of their misconceptions and misunderstandings when they know that the risks they take to express their misguided ideas will not be used to ridicule them, but will be used to empower

them for the next stage of learning and that their grades are a reflection of the empowerment they have and still need.

Assessment and Grading

The National Council of Teachers of Mathematics' *Assessment Standards for School Mathematics* (NCTM 1995) provides guidelines for our facilitation of assessment in all areas of school mathematics, one of which is geometry. According to NCTM, "using multiple sources of evidence [assessment techniques] can improve the validity of the inferences made about students' learning. The use of multiple sources allows strengths in one source to compensate for weaknesses in others" (pg. 19). Varied assessment techniques empower the teacher to examine students' knowledge and understanding from different perspectives and at different levels. This position is repeated and further supported in the NCTM' *Principles and Standards for School Mathematics* (NCTM, 2000): "To make effective [instruction and curriculum] decisions, teachers should look for convergence of evidence from different [assessment] sources. Because different students show what they know and can do in different ways, assessments should allow for multiple approaches, thus giving a well-rounded picture and allowing each student to show his or her best strengths. (pg. 23).

Topics in mathematics lend themselves well to a variety of assessment techniques. Geometry, the study of shapes and relationships between their attributes and solids and the relationships between their attributes, is a topic that engages learners from many different approaches (e.g., hands-on, abstract, proof, visualization). The broadness and depth of this mathematical subject make it appropriate and necessary to use multiple sources to assess students' understandings of the subject. Six sources or techniques of assessment that can be used to obtain information about students' geometry learning are (1) observations, (2) writing, (3) portfolios, (4) interviews, (5) demonstration/performance tasks, and (6) written tests and quizzes. This is not a complete list, but just a platform from which a teacher can create assessment plans to determine how and what students are learning in geometry.

The focus on assessment is to obtain information about students' learning so that one can make changes and improvements in curriculum and instruction to support students' learning. Assessment is also the means by which data can be collected and used for grading, the act of assigning letter rankings or numerical scores to students' work according to a predetermined standard. One system for grading involves the use of rubrics, a scale that accountants for various components of assessment data and their value according to standards and expectations for student learning. Rubrics represent holistic grading, that is grading that gives attention to process as well as product and to the bigger picture of what students are able to do and do know although the final outcome might not be the perfect, right, or exact answer. When appropriate, a rubric for assigning grades for the assessment techniques is suggested. An example of a percent that each of the assessment methods might have in the students' overall grade are presented in Table 1. Modifications to these suggestions are encouraged, as student populations are diverse in their learning potential, needs, and styles.

Table 1. Example Percent Value for Each Assessment Technique

Assessment Technique	**Percent Value**
Observations	20%
Writing	20%
Portfolios	20%
Interviews	5%
Performance Projects	20%
Written Tests & Quizzes	15%
Total	100%

Observations

Observation is one of the most common means by which we collect data about almost anything in the environment. Because geometry, plane and solid, is a "doing" subject, it is evident that a great deal of assessment information and grading data will be collected when students are "doing" geometry. The level and purpose of observations will vary with grade level. In the primary

grades, observation of students' activities and behaviors is a dominant form of assessment, because young children are mostly exhibiting what they know through what they are saying and activities they are participating in, more than by what they make write or demonstrate with geometry tools (e.g., compass). This is not to say that observation is less valued in the intermediate or upper grades. Observation is a valuable assessment technique that can be implemented by teachers at all grade levels.

When students are engaged in geometry learning, there are myriad things that can be observed and noted as a means of assessment. The observation form in Figure 1 might be used to observe the whole class, a small group of students, or an individual student at work. The form can be used to collect frequencies and notes of students during learning experiences. The form can be modified for use as a peer observation tool or a self-evaluation tool, thus directly involving students in the assessment of their learning. When using observations as a means of assessment, there are four things to keep in mind:

1. Observations during students' geometry experiences, in most instances, involve the senses of seeing, hearing, and touching. You can see what the students are doing, how they are doing what they are doing, the results of what they are doing, and how they are responding to what they are engaged in and what is happening around them. You can listen to students as they engage in oral discourse during group and cooperative learning activities. You can also listen to the kinds of statements students make and questions they ask of one another. Students' conversations with one another often reveal understandings and misunderstandings that are common throughout the class and can be subtly or explicitly addressed during whole-class instruction.

Figure 1. Sample Observation Assessment Form

Geometry Observations	**Grading (36 points maximum)**
1. Identifies shapes and/or solids by informal or formal name	3: consistently

(examples: box/cube, rooftop/triangular prism) **Notes:**	2: sometimes 1: rarely 0: never
2. Reproduces and/or constructs shapes and/or solids (examples: draws a pentagon, builds a prism, cuts out a triangle) **Notes:**	3: consistently 2: sometimes 1: rarely 0: never
3. Engages in meaningful discourse with classmates (example: discusses characteristics of shapes and/or solids) **Notes:**	3: consistently 2: sometimes 1: rarely 0: never
4. Uses manipulatives appropriately (example: models shapes on geoboard and with pattern blocks) **Notes:**	3: consistently 2: sometimes 1: rarely 0: never
5. Classifies shapes and/or solids (example: classifies triangles by length of sides) **Notes:**	3: consistently 2: sometimes 1: rarely 0: never
6. Describes shapes and/or solids (example: describes a cylinder in writing and orally) **Notes:**	3: consistently 2: sometimes 1: rarely 0: never
7. Identifies shapes and/or solids in the environment (example: suggests doorway models a rectangle) **Notes:**	3: consistently 2: sometimes 1: rarely 0: never
8. Uses learning tools appropriately (example: uses a mirror to find lines of symmetry) **Notes:**	3: consistently 2: sometimes 1: rarely 0: never
9. Compares shapes and/or solids (example: describes differences between a cone and a cylinder) **Notes:**	3: consistently 2: sometimes 1: rarely 0: never
10. Uses appropriate geometrical terminology (example: applies definitions accurately) **Notes:**	3: consistently 2: sometimes 1: rarely 0: never
11. Describes/models relationships among shapes and/or solids (example: use Venn Diagrams to display relationships) **Notes:**	3: consistently 2: sometimes 1: rarely 0: never
12. Shows interest in geometry explorations (example: engages in specific individual or group assignment) **Notes:**	3: consistently 2: sometimes 1: rarely 0: never

Here's an example of discourse among fourth graders that provides rich information regarding students' understanding of the measure of angles in a triangle:

Student 1: "What we have to do is to show that the angles of a triangle all add up to 180 degrees."

Student 2: "What triangles?"

Student 1: "All triangles or any triangle we make up."

Student 2: "How can we make up a triangle that works?"

Student 3: "Whatever we make up [triangle] has to work."

Student 2: "How can we tell that the angles add up to 180 degrees if we don't know what the angles are?"

Student 1: "That's what we have to do."

Student 2: "So if each angle has some degrees but we don't know what they are but we have to show that they add up to 180 degrees, I say we make up some degrees and draw them in a triangle and then we will have it."

Student 3: "I don't think that will work 'cause you don't really know what the angles are so you just can't make up numbers. What we can do is get one of those things to make angles [protractor] and make some angles and put those together to make a triangle and if they add up to 180 degrees then we have the answer."

Student 1: "Yeah, let's do that." (Student 3 walks away to retrieve a protractor.)

Teacher: "How's it going in this group?"

Student 1: "We getting ready to get some angles and put them together to make a triangle and if they fit into a triangle, we will add them up and if they add up to 180 degrees then we know we have the right answer."

Teacher: "Let's think about this. You know what an angle is right?

Students (unison): "Yeah."

Teacher: "You know what a triangle is right?"

Students (unison): "Yeah."

Teacher: "And we have already used protractors to show that the sum of the angles in triangles is 180 degrees. So that you know: the sum of the angles in a triangle is 180 degrees, so you don't need to find angles with a protractor and put the angles together to make a triangle. You would only be doing something over you already know. What I want you to do is prove that the sum of the angles is 180 degrees even if you don't know the measurement for each angle. I'll give you a hint: Think about something you already know that is 180 degrees or just find something that measures 180 degrees and see if you can make a connection."

Student 2: "OK, but then what do we do?"

Teacher: "I'll let your group talk about it some more, and I'll check on you again in a few minutes."

Student 2: "I think this is harder now."

(Student 3 returns. Student 1 and Student 2 share the teacher's comments with Student 3.)

Student 1: (To Student 3) "We don't need that thing now. We have to do something else, 'cause we can't know what the angles measure."

2. Decide what things you might want to observe before beginning the observation. Many items might come to mind during the observation. However, if you start with a plan, you will be more focused during the observation. In addition, planning ahead will prepare you for questions students might have about and during the observations.
3. Make notes during or immediately after the observations. Sometimes important information about students' learning may be lost when time wears on one's memory. You can (a) use a form like the example provided, (b) use note cards, (c) make voice recordings into a hand-held tape recorder, and/or (d) use a well-stationed video recorder.
4. As the classroom teacher, you are a participant observer: involved in the learning experience but retaining exterior motives for that involvement. As a natural part of the learning community, your observations can be an unnoticeable part of students' learning experiences. Being engaged as a learner in some assignments will enable you to make observations as a learner as well as a teacher and perhaps see needed changes in assignments that you did not see from the teacher's perspective.

Writing

Writing in mathematics is an excellent way to obtain assessment information about students' learning of geometry. Assessment of students' writing helps to make inferences about students' (1) use of vocabulary and definitions, (2) understandings of relationships between concepts, and (3)

expressions of visualizations. Grading students' writing in geometry is not a difficult task if a good system is in place. Once students are writing about and with geometry, a rich source of grading data results from the students' products. Writing assessment data (even "picture" writing by younger children) provides inferences about what students are thinking and how well they are connecting geometrical concepts. Figure 2 shows an assessment format that might be used for a writing assignment where students have been given a list of geometrical terms and phrases and have been instructed to write a narrative (e.g., geometry short story, poem) using all of the terms and phrases.

Figure 2. Sample Writing Assessment Form: Terms & Phrases Narrative

Components	**Grading (12 points maximum)**
Part 1: Directions	3: Used all terms and phrases 2: Used some terms and phrases, but not all 0: Did not use any terms and phrases
Part 2: Definitions	3: Used all terms and phrases correctly 2: Used some terms and phrases correctly, but not all 0: Did not use any terms and phrases correctly
Part 3: Synthesis	3: Presented accurate and appropriate relationships between terms and phrases that contribute to a sense of cohesiveness in the narrative 2: Presented accurate and appropriate relationships between terms and phrases that contribute to a sense of cohesiveness for only a portion of the narrative 0: Did not present accurate nor appropriate relationships between terms and phrases that contributed to the narrative, that is, statements are unrelated or isolated and do not come together to form a cohesive narrative
Part 4: Creativity	3: Presented a narrative that included creativity in regards to context, plot, characters, real-life situations, fantasy, mystery, etc. 1: Presented a narrative that use the terms and phrases in a sterile manner, without interjection of creativity

However getting students to engage in a writing task in mathematics is often very challenging. Listed below are five creative ways to entice students to write about and with geometry.

1. Assign a shape or solid to each student. The student is to take on the identity of the shape or solid and write a poem or a short story related to that identity.
2. Ask students to write the directions to their home using geometrical terminology to describe landmarks (e.g., signs, buildings). They can also produce a map of the route to accompany the directions that includes outlines of various landmarks.
3. Ask students to develop "Who am I?" riddles and exchange them with each other (e.g., I am a shape with only three sides and two of my sides have the same length. Who am I?)
4. After studying geometrical concepts and skills, give each student a blank index card. Ask each student to respond to various prompts (e.g., Today, I learned that . . . , I wish I knew what . . .)
5. Students can develop a geometry encyclopedia for the classroom. Each student can research a geometry concept and add their drawings (or photographs) and descriptions to the classroom encyclopedia.

Portfolios

Portfolios, collections of students' work (e.g., drawings, constructions, writings, photographs) can provide valuable information about students' geometry learning. In particular, portfolios can be assessed to acquire information about students' learning from one period of time to another, such as before, during, and after a long-term unit of study. What is important is not the number of items students include in their portfolios, but the learning that is represented by the items. In Figure 3 are some suggestions for portfolio contents.

Figure 3. Portfolio Description

The Geometry Portfolio

• Drawings and models of two-dimensional shapes

• Models of three-dimensional solids

• Descriptions of shapes and solids

• Report of in-class and out-of-class investigations and explorations

• List of geometry terms and definitions learned and evidence of accurate application

• Questions still unanswered by instruction

• One or more items chosen by the student to represent /his her best work

• Evidence to show exploration of relationships between shapes, between solids, and between shapes and solids (e.g., concept map)

• Evidence of explorations with formulas (e.g., relationship between area and perimeter of a shape – how changes in one affect the other)

• Results of geometry problem solving explorations (process & product)

• Data collection on geometry in the environment (e.g., sidewalk cut-outs in the form of squares, refrigerators are prisms)

• Hands-on geometry proofs (e.g., 180 degrees in all triangles by cutting off angles and making a straight line of 180 degrees)

• Geometry quiz or exam (previously taken) with corrections made for inaccurate responses

Portfolio contents may have already been through the grading process or some content may be for information only, and thus only some items would receive grades after submission of the portfolio. However, one may also acknowledge the portfolio as one comprehensive package and grade it in its entirety. A rubric for grading the complete portfolio is presented in Figure 4.

Figure 4. Sample Portfolio Assessment Form

4: Exceeded expectations (e.g., gave extra care to presentation, included components not required, understanding exhibited)
3: Met expectations (e.g., followed directions, met minimum requirements for category, understanding exhibited)
2: More effort needed (e.g., work incomplete, errors in work, understanding in progress)
1: Resubmit item (e.g., too much incompleteness, too many errors, misunderstanding and misconceptions apparent)
NG: Not graded (e.g., previously graded, grading not appropriate)
(64 point maximum)

Contents	**4**	**3**	**2**	**1**	**NG**
❖ Drawings and models of two-dimensional shapes					
❖ Models of three-dimensional solids					
❖ Descriptions of shapes and solids					
❖ Report of in-class and out-of-class investigations and explorations					
❖ terms and definitions learned and evidence of accurate application					
❖ Questions still unanswered by instruction					
❖ One or more items chosen by the student to represent /his her best work					
❖ Evidence: exploration of relationships between shapes, between solids, and between shapes and solids (e.g., concept map)					
❖ Evidence: explorations with formulas (e.g., relationship between area and perimeter of a shape – how changes in one affect the other)					
❖ Results of geometry problem solving explorations (process & product)					
❖ Data collection on geometry in the environment (e.g., sidewalk cut-outs in the form of squares, refrigerators are prisms)					
❖ Hands-on geometry proofs (e.g., 180 degrees in all triangles by cutting off angles and making a straight line of 180 degrees)					
❖ Geometry quiz or exam (previously taken) with corrections made for ❖ inaccurate responses					
❖ Miscellaneous Item(s)					
Totals					

Interviews

Interviews in the classroom can take place in at least three points in time: (1) before instruction (diagnostic), during instruction (formative), and after instruction (summative). Interviews may be conducted with an individual student or with a small group of students. Interviews may be unstructured (questions and discussion prompts not preplanned) or structured (questions and discussion prompts planned to guide the interviews). The most important thing is for the teacher to have one or more purposes for the interview and to know what kind of information is important to gather during the interview. The interviews may involve discourse only or may also involve the students' engagement in some sort of demonstration or modeling and responding to questions. Figure 5 presents an example of a structured interview that may be used to acquire information about students' classification of triangles. The interview may be modified for other geometry topics and may be used with more than one student at a time. Professional and personal preference can be used to determine whether or not grading interviews is appropriate for the students being served. It is advisable to consider interviews as information only activities as to not create tension in conversations and discourse with students at other times during instruction. However, students may receive credit for appropriate participation in the interview.

Figure 5: Sample Geometry Interview Guide

GEOMETRY INTERVIEW: TRIANGLES

Teacher has a variety of triangular models (same color) lying on the table or displayed on a computer screen:

1. Have you seen these shapes before? If so, where?
2. What do you know about these shapes?
3. What do the shapes presented here have in common?
4. What are these shapes called? How do you know?
5. How are these shapes different from the shape of the table or the computer screen?
6. Sort the shapes into two or more groups. Why did you sort the shapes that way?
7. Sort the shapes again into two or more groups. Why did you sort the shapes that way?
8. Are there any shapes that are exactly the same? Can you cover one shape completely (e.g., perfect fit) with just one other shape? Do you know what word we use to describe such shapes?
9. Can you put two or more of the shapes together to make a new shape? What new shapes can you make?
10. What kind of line symmetry does each shape have (e.g., none, horizontal, vertical, diagonal)?
11. Name some other shapes that are not like these shapes? How are they different?
12. What comments would you like to add or questions would you like to ask?

Notes:

<u>Demonstrations/Performance Tasks</u>

Regardless of the assessment technique used, geometry from the hands-on perspective is a must. The hands-on perspective supports the use of demonstrations and performance tasks as a means of collecting assessment data. Students can demonstrate such things as the proper use of learning tools and manipulatives. For example, some students may be adept at paper folding (origami) and will be able to demonstrate how to make various shapes and solids.

There are many performance tasks that can be implemented to guide students in applying a variety of concepts and skills. One task is to request students to build a solid using everyday objects such as straws, pipe cleaners, lollypop or Popsicle sticks, index cards, string, pencils, etc. Students can display their productions on mobiles in the classroom and engage in discussions about the variations of the objects. They can also test characteristics, such as strength, of the model solids.

Another useful performance task is to instruct students on how to build scaled models of objects or buildings (e.g., the United States' Pentagon, the Egyptian pyramid at Giza, etc.). By using a scale like one inch represents 100 feet (or whatever is most appropriate). A performance task such as this encompasses plain and solid geometry, measurement, number sense, visualization, whole numbers, and fractions. In addition, a building project such as this can integrate mathematics with other subjects such as social studies and history.

On the day(s) for demonstrations and performances, students can set up their own stations. One-half of the class plays the role of audience for the first part of the instruction time and one-half of the class plays the role of demonstrators/performers. This is reversed for the second part of the instruction time. Small groups of students and the teacher can rotate from station to station viewing and observing the demonstrators/performers at work. Each demonstrator/performer might also have a visual display (e.g., poster) informing the audience of the subject and expectations of the project. Students in the audience can ask questions and make comments to the demonstrator/performer. The teacher might also choose to photograph the students at work and include this and other records of the task in the students' portfolio for further assessment. After all of the demonstrations and

performances are complete, whole class discussions can be initiated to solicit students' reactions to and understanding of what was observed in the experience. As the students share their experiences, more information about what and how they are learning can be gathered to assess the students' understanding of the geometry involved in their project.

During individual or group sessions, students can demonstrate geometrical visualization skills by completing tangram puzzles (e.g., placing the seven trangrams shapes in outlines to create a noticeable shape), cutting out nets or patterns for three-dimensional solids, and manipulating shapes to show relationships (e.g., cutting off a triangle from a parallelogram (non-rectangle) and moving the piece to create a rectangle to show the relationship between the area formulas of the two shapes).

In addition to teacher-suggested demonstration and performance projects, students can also design their own projects to show how and what they are learning in geometry. These student-centered projects offer great opportunities to students to develop and exercise interest in geometry and study particular segments of the topic that are of interest and meaning to the students. The form given in Figure 6 can be modified to accompany specific oral and visual presentations students might engage in for geometry learning.

Figure 6. Demonstration/Performance Assessment Form: Shapes in Nature Presentation

Components	Grading (10 points maximum)
Part 1: Group Member Participation (Four students assigned to a group)	2: All group members participated 1: All group members did not participate 0: No group members participated
Part 2: Presentation Topic (Oral) (One or more speakers)	2: Topic focused on shapes in nature and from various places in nature 1: Topic focused on shapes, but not in nature or from limited places in nature 0: Topic did not focus on shapes
Part 3: Presentation Topic (Visual) (Some type of visual display required)	2: Topic focused on shapes in nature and from various places in nature, shape concepts correct 1: Topic focused on shapes, but not in nature or to limited places in nature, shape concepts correct 0: Topic did not focus on shapes, shape concepts not correct
Part 4: Delivery (Group had time to prepare and practice for oral delivery)	2: Well Organized, within time limit, connected to audience 1: Poorly organized, over time limit, weak connection to audience, 0: No organization, no attention to time limit, no connection to audience
Part 5: Visual Display (Could be drawings, photographs, pictures from magazines, etc.)	2: Organized, labeled, neat 1: Poorly organized, items not labeled, display not neat 0: No visual display

Written Tests and Quizzes

The use of written tests and quizzes is a traditional, but nonetheless a valid means of collecting assessment data related to students learning of geometry. There are many ways to design written tests and quizzes so that they represent ways of dealing with diverse learners. One way is to use a variety of formats for the items of the tests and quizzes. Figure 7 lists several different formats

for items on written tests and quizzes; formats that result in authentic information about what students do and do not know.

Figure 7. Item Formats for Written Tests and Quizzes

True/False with Explanation

Example: All squares are rhombi. True or False. Explain.

Short Answer

Example: Describe the similarities and differences between cubes, cylinders, cones, and prisms.

Fill-in the Blank

Example: Two triangles that have the same size and the same shape are called _____ triangles.

Matching

Example: Match (draw connecting lines) the following solids with the proper characteristic:

Sphere	Has eight edges
Square-based Pyramid	Has no sides
Cube	Opposite sides are parallel

Drawing/Sketching

Example: Make a drawing or sketch of the following shapes:

Regular pentagon, Trapezoid, Rhombus, Parallelogram

Conclusion

What has been presented here are just a few assessment techniques that teachers can use to gather information about what and how students are learning in geometry. Using multiple sources to collect assessment data is important to insure that diversity of students' learning styles and differing strengths of assessment techniques are accounted for in instruction and curriculum. Observations, interviews, writing, portfolios, demonstration/performance tasks, and written tests and quizzes are just several ways students can be assessed when studying geometry. In trying the techniques presented in this article, one will probably find that some techniques are more suitable for various students, for various topics, and at various times in the curriculum and instruction program. The overall aim of assessment is to obtain information that will empower teachers to create classroom experiences that will enhance and improve students' learning.

References

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