Reform and Traditional Undergraduate Calculus I:

A Meta-analysis

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Abstract

This study is a meta-analysis of eight studies of undergraduate Calculus I. The hypothesis tested is whether calculus reform has had a positive impact on undergraduate Calculus I students' conceptual understanding of calculus. According to the data in this study, the reform teaching techniques increased undergraduate Calculus I students' conceptual understanding.

Starting in the early 1980's, the reform calculus movement continues to occur in college-level calculus. Many students leave high school having taken courses in geometry, pre-calculus and calculus and in addition, they have had substantial exposure to technology. "New Math" and the refocus of high school mathematics on new standards were two driving forces in the college calculus reform movement. The "New Math" movement was "initially motivated by a desire to update school curriculum in a balanced way, emphasizing set theory as much for its use in probability as for its role in theory, the 'New Math' came to be oriented overwhelmingly towards theoretical foundations, with an absence of applications" (Leitzel and Tucker, p. 9). A low number of students were successfully completing calculus and even when students did complete calculus they had little understanding of what it was they were learning. Students could run algorithms and apply formulas, but had trouble with application of the calculus they were learning. A frustrated calculus faculty was upset by the unmotivated and poorly prepared students who had previously taken calculus. Calculus was being used as a filter and not a foundational course by some subject areas that never used calculus. Mathematics in general was lagging behind other subjects in its use of technology. (Leitzel and Tucker, 1994)

Have these driving forces of the reform calculus movement caused a change in the students' learning? The research question is: “Does reform calculus improve students' conceptual understanding of calculus.” A universal definition of reform calculus has not yet been established and perhaps never will. The definition used in this study will be:

Definition 1*: Reform Calculus is teaching that has a change in instruction mode and/or implementation and integration of technology*.

Change in instruction can include moving away from a lecture only structure, such as using group work. We agree with the statement that “how calculus is taught has changed more than what is taught" (Leitzel and Tucker, p. 1). Change in instruction and technology are the two prominent changes that address this shift of emphasis. The reduced emphasis on symbolic manipulation allows more time to be spent on what is commonly known as the rule of four: calculus from a graphical, symbolic, numerical, and verbal perspective. Less time is spent on symbolic manipulation so that there is time to spend on the remaining perspectives. The increased use and implementation of technology can be seen in both the use of the graphing calculators and computers. This usage fluctuates from a daily computer lab to once a week usage with the latter being the most prevalent. In some cases, the weekly use and integration of the calculator and/or computer lab replaces lecture or recitation (Ganter, 1997).

This study looks specifically at how reform calculus affects students' conceptual understanding. Similar to the definition of reform calculus, many handwave their way around a concrete definition of conceptual understanding**.**

Definition 2*: Conceptual understanding is a student's ability to apply and connect what he or she is currently learning to past information, future mathematical learning, and other related fields.*

Students that conceptually understand a topic know the concepts involved and they take ownership of what they know. These students are able to see the big picture. Conceptual knowledge is characterized most clearly as knowledge that is rich in relationships. It can be thought of as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information. (Hiebert and Lefevre, 1986, pp. 3-4) The ability to take a step back, look at the overall picture, and see how everything is related and connected is conceptual understanding.

Meta-analysis

A meta-analysis is a statistical process for summarizing the quantitative results from the studies that address the same research question under similar conditions. Instead of running a new study, the data used are studies that have already been done. To do a meta-analysis, a research question is asked and a hypothesis formed. In this project, a meta-analysis was conducted in order to analyze the hypotheses: The calculus reform movement has had no impact on undergraduate Calculus I students' conceptual understanding of calculus knowledge. The alternative hypothesis is that the calculus reform movement has had a positive impact on undergraduate Calculus I students' conceptual understanding of calculus knowledge. The next step is to collect all of the relevant studies done on the topic begin tested. Important statistics and data are collected from these studies and then a statistical analysis can be run. As Fredric Wolf states: "Each data point used for analysis is obtained form an individual study rather than from an individual subject, as typically done in a traditional research study" (Wolf, 1986, p. 11). The end result is an overall effect of all studies examining the original question that is asked. When there are numerous studies done on a topic of interest, a meta-analysis is a much stronger tool than running a new study.

We followed the procedural steps given in McNamara et al. (1998) in conducting our meta-analysis. The initial fear of running a meta-analysis on the topic of reform calculus versus traditional calculus was being able to find enough studies. The number of articles on reform calculus is abundant, but the number of actual studies was in question. A complete computerized literature search was done and it located 38 research studies. In the computerized literature search, the databases used were Education Resource Information Center, Dissertation Abstracts, and Education Index. Key terms that produced results were calculus, reform, and research. Reading each citation's abstract about the article allowed for filtering. Articles immediately removed were ones that were informative in nature, but were not experimental studies with data that we could quantify. Two researchers read the remaining articles. After reading the articles, they discussed and agreed on which articles should be removed from the study because the study did not meet the reform calculus definition or the conceptual understanding definition (or both) that we were using. To be sure that the search was complete, the compiled studies' bibliographies were searched to find any studies that were missing. This process of searching for relevant studies continued until the bibliographies of the studies being reviewed no longer produced new studies. In the end, eight studies met our criteria.

In a meta-analysis, it is important to identify the independent variables, dependent variables, target population, time period, and research design. The independent variables are reform calculus courses; i.e. courses that have had a change in instruction mode and/or implementation and integration of technology. This includes classrooms that use a graphing calculator or a computer algebra system. It also includes those classes that use a reform textbook or increase student interactions through group work or discussions. The dependent variable of this study is conceptual knowledge of calculus. The target population is students that are in undergraduate Calculus I. The time period that is being studied is from 1980 to the present. Reform calculus as we have defined it has occurred in the last 20 years and, therefore, we did not look at any studies earlier than 1980.

The research design includes changes that have been done: the reform, the outcome measures, and the testing that has been done to test the hypothesis. As McNamara et al. explain, “for many meta-analysts, this is the most critical step because it describes the complete instructional intervention that should be replicated to yield similar results” (1998, p. 384). All of the studies have a common research goal, which is to examine the effects of reform calculus. Classrooms studied have implemented and evaluated a reform instructional program. It is here that the studies begin to differ. The programs examined have all implemented reform calculus, but each program is unique.

It is important to reiterate the definition of reform calculus used in this study. The definition used in this project is calculus that has a change in instruction mode and/or classrooms that have integrated technology. Calculus courses that have an add-on computer or calculator assignment or project will not be included. Also excluded are classrooms that only put conceptual problems on homework without changing teaching style or mode. These last two examples are not in the spirit of reform calculus as we have defined it. Some may disagree, but this is a judgment call. In our opinion, reform calculus must embrace change and new approaches.

As for the outcome measures, conceptual understanding is being examined. Again, conceptual understanding is a student’s ability to apply and connect what he or she is currently learning to past information, future mathematical learning, and other related fields. Students that conceptually understand a topic truly appreciate what it means and they take ownership of what they know. It is the ability to see the big picture.

Lastly, we examine what was done at the end of the instructional program to test the hypothesis. Was a final exam given? Were exams looked at throughout the semester? How did each researcher run his or her study? Each study, although not identical in their methods, has the common goal in the method of testing reform calculus.

Included Studies

With the above outline of meta-analysis, we give a short description of the included studies. The studies are in alphabetical order by author. We will begin with a study by Beckmann (1998) which examined four different courses. We use two of the courses in the meta-analysis. In the reform course, students studied content that was graphically developed and of a conceptual nature. The course itself was dependent on the use of computer graphics, and the students used computer graphics software to complete supplemental assignments. In the traditional course, skill development was emphasized without the use of graphical software. Test and quiz results as well as attitude survey results were used to find that the reform section had higher understandings and interest, without lower skill level.

Brunnet's (1995) reform section emphasized concepts. The Calculus Consortium Based at Harvard textbook was used and students participated in a weekly lab. This lab included group work and real world problems. The control section consisted of students in the traditional sections. In this study, the two groups were given items from a placement test that was originally published by the Mathematical Association of America. The traditional section outperformed the reform section on these items.

The next study included was done by Cooley (1995) where students in the reform class had a weekly session in the computer lab. In the lab, they did activities and assignments that were intended to enhance their conceptual understanding. The labs included were written for *Mathematica*. The control class did not have a computer session each week, but instead, had a second recitation class. Based on final exam results and the results of a researcher-written conceptual exam, it was found that the reform class did better on both conceptual and traditional calculus questions, including computational questions.

For the study done by Cunningham (1991), the two groups considered were similar in nature, except for the amount of technology used. The control group was only exposed to the computer during classroom demonstrations. The treatment group had classroom demonstrations and also had outside access to the software for homework and as a study aid. The experimental group frequently saw things done on the computer before they learned how to solve the problems by hand. These students were allowed to use the computer on exams. Based on results from giving the same tests to each group, it was concluded that no significant differences were found in conceptual understanding between the two groups.

Two groups were examined in the Garner (1998) study. The experimental course made use of the Calculus Consortium Based at Harvard curriculum and text. This section had more activities and more student interactions. It emphasized conceptual skills and application problems. The idea was to lead students gradually through calculus ideas and give student more time to develop reasoning. The control group used *Calculus and Its Applications* by Goldstein, Lay, and Schneider (1996). It presents the theoretical ideas first and applications later. It was concluded that no significant difference between reform and traditional was found on the written test mean scores. With that said, they thought that there were differences in the students retained knowledge. “The significant difference between the groups in mean conceptual and procedural scores is evidence of this fact… it appears that the reformed course gained in conceptual understanding when compared to the traditional course. But they did not acquire or retain the same procedural ability”(Garner, 1998, p. 108).

In the Heid (1988) study, the traditional section had two 75-minute lectures each week and one 50-minute recitation section in which calculus skills and algorithms were studied. The reform section had three 50-minute class periods and spent time on the traditional skills. In the reform section a number of approaches were used to teach calculus concepts and ideas. The reform section also made use of microcomputers and the students’ homework included microcomputer assignments. It was found that the students in the experimental class did better on conceptually oriented questions and they discussed the calculus concepts in more detail. On the final exam, the experimental students performed almost as well as the comparison class.

In the study by Hurley, Koehn, and Ganter (1999), seven semesters of data was included. The reform class had a computer lab day replace one-classroom hour and turned a recitation session into a group problem solving session. In these group problem sessions, students self-selected their groups and worked on problems that were both conceptual and computational. The other two class periods were the traditional lecture sessions where students “often worked together (with the aid of graphing calculators) to analyze and investigate problems or properties and suggest how the instructor should proceed”(Hurley, Koehn, and Ganter, 1999, p. 801). They found that the reform students did better than the traditional students when given the same final exam.

The last study included was done by Keller and Russell (1997). The reform section was taught using the TI-92 and frequently required *Mathematica* projects. They participated in written labs and worked in groups of 3 to 4 students. The traditional students did not use technology. It was found that the reform students did better than the traditional students when tested using the departmental two-hour comprehensive final.

Results

To test our null hypothesis a confidence bound was formed. See Chart I, which summarizes the important statistics. The number of students involved in the study is given by v. The estimator of the study effect size is given by *d* and d^U is the unbiased estimator. Hedge's *g*, a variation of Glass's *d*, is the main statistic used for *d*. (Glass, 1976; Hedges, 1981) Fredrik Wolf describes *d* well, "The goal is to obtain a pure number, one free of our original measurement unit with which to index what can be alternatively called the degree of departure from the null hypothesis of the alternative hypothesis or the ES (effect size) we wish to detect"(Wolf, 1986, p.24).

Author	v	d	d^U
Beckmann	54	-0.528	-0.5206
Brunett	70	-0.6761	-0.6688
Cooley	120	0.489	0.4859
Cunningham	51	0.7126	0.7021
Garner	106	0.4697	0.4664
Heid	14	0.35	0.3309
Hurley, Koehn, &Ganter	239	0.278	0.277
Keller& Russell	267	1.224	1.221

Chart I: Summary of Meta-analysis Statistics

Testing the Null Hypothesis

Now we are able to test our null hypothesis (the calculus reform movement has had no impact on undergraduate Calculus I students' conceptual understanding of calculus knowledge) against the alternative hypothesis (the calculus reform movement has had a positive impact on undergraduate Calculus I students' conceptual understanding of calculus knowledge). A 97.5% confidence bound was formed using the unbiased[i] estimator $D^U = \frac{1}{k}\sum_{i=1}^{k} d_i^U$. The 97.5% confidence bound is $(0.09957, \quad \infty)$. Because zero does not belong in the above interval, we reject the null hypothesis.

In the last confidence interval, we assumed that each study had the same mean. "Conceptually, we assume that each experiment to be combined is a replication of the others, differing only in the response scale and sample size...Thus, δ, the standardized mean difference, is the treatment effect (mean difference) when the response scale has unit variance. The parameter δ is called the effect size."(Hedges, 1981, p.108) We can also run this test if we do not assume that $\delta_i = \delta$, that is, each study had the same mean or effect size. To do this, $\hat{\delta}_i = d_i^U$, and the new 97.5% confidence bound is as follows $(0.09756, \quad \infty)$. This also results in a rejection of the null hypothesis.

Limitations of the study

Of course, there are limitations in any meta-analysis and ours is not an exception. The included studies were based on our definitions of reform calculus and conceptual understanding. Others may disagree with our definitions.

Even with agreement on our definitions, others may disagree with the particular studies we included and excluded. Perhaps we used data from a low quality study (although we tend to think this is not true with the particular studies we used). It is likely, however, that there is missing data, including studies that were never made available to the public.

Another limitation is that the results are based on the outcome measures and not as much on interaction effects. To account for this in future research, coding characteristics can be used to see if there are in fact interaction characteristics and effects. This is difficult to do as "there is no systematic logical procedure to identify these characteristics"(Wolf, 1986, p. 53). The statistical theory that is used in a meta-analysis has its own limitations.

Conclusion

The research literature was searched to find studies of undergraduate students' conceptual understanding of calculus when calculus was presented as reform calculus**.** Eight studies were identified based on our definitions of reform calculus and conceptual understanding. A meta-analysis was conducted and the statistical hypothesis of no increase (versus positive increase) of students' conceptual understanding under reform calculus was tested. According to the data in this study, the reform teaching techniques increased undergraduate Calculus I students' conceptual understanding.

References

*References marked with an asterisk indicate studies included in the meta-analysis.

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[i] We need *d* to be unbiased because when *m* (the same size) is small, there can be substantial bias. Bias can cause the estimator to overestimate δ, therefore we want to get a modified estimator that is unbiased (Hedges, 1981).