Ask the Write Question!

Pat Mower

Over the last decade of teaching college mathematics, I have toyed with the lofty notion that there must be a way to get students to think mathematically outside the cube. I want my students to "see" mathematics as art…to question why, not just how the algorithm works…to ponder if the concept in question was discovered or invented…. to experience the "aha" earlier than the next course!

To this end I have experimented with many activities…writing-to learn activities, peer teaching activities, the creation of mathematical fiction, group board work, role playing, peer reviewing and tutoring experiences, math games; all with a limited amount of success. The questions posed below are part of one of my more successful attempts.

The idea is this: ask the students to ponder a question of depth and weight while they are working practice problems related to a particular mathematical concept. I assume we all agree that some drill is vital to the learning process. Moreover, I contend that the thinking ,composing, creating, and writing of a solution to a more abstract question regarding the concept is equally important. I once heard that moments of genius come after a period of conscious study and hard work followed by a period of rest where the subconscious continues working, then upon arising, "aha!" Thus, the idea is invented (or discovered - if you are the more spiritual mathematician.)

Okay, so imagining my students having moments of genius is a little over the top. But, encouraging them to communicate their thoughts regarding the concept at hand seemed highly plausible. Therefore, I began asking my mathematics students to communicate with me regarding the mathematical concepts we explored in class.

Communication of mathematical ideas is already a vital component of most K-12 mathematics programs. The National Council of Teachers of Mathematics addresses the necessity of this communication in its Principles and Standards for School Mathematics:

Instructional programs…should enable all students to –

- Organize and consolidate their mathematical thinking through communication.
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely (NCTM, pg. 348).

I contend that college mathematics students should also be required to communicate mathematics effectively.

The write questions actually began as part of an online college algebra course I oversaw one semester. My goal was to encourage the online students to respond electronically in words to questions on content so I could see if they understood. Soon I realized we needed some ground rules. The following rules have evolved, but remain short and sweet so the student remembers.

Write question rules:

1. The written response must contain at least three complete sentences.
2. Be COMPLETE (use the major points from the class discussions and text), CONSISTENT (do not contradict yourself), and CLEAR (write as if you were explaining your response to an absent student.)

3. Examples and graphs are allowed to clarify written ideas.

4. The response may be typed or clearly hand written.

5. The grade will be determined by how well the above rules are followed and by the accuracy and creativity of the response.

The first write question assigned for a College Algebra course (regular classroom format) one semester was:

Which is larger the set of Integers or the set of Real Numbers? Defend your answer.

We had not discussed the cardinality of infinite sets in the course, so the answers varied and were somewhat subjective. Most students cited the set of reals as the larger set. One student wrote “Real nos. because it has fractions.” Two students referred to the notion of a one-to-one correspondence with the natural numbers in their arguments. My assessment of and comments on their written responses of this initial question allowed the students to see what I was looking for in a response.

The following write questions were composed for the same College Algebra class and corresponded to a chapter titled Linear Functions. This set of questions targeted the concepts discussed in the daily lessons. After some of the questions are comments and samples of students’ good and not-so-good responses, highlighted by quotations marks. My observations regarding individual responses are given in brackets.

1. **Describe how to solve the linear equation: ax + b = 0. Assume a and b are positive integers.**

“ax + b = 0 can be written h(x) = ax + b so you could use the x intercept method to solve. Put the equation on a graph and find the point where the x intercepts the x axis. The “x” coordinate of that point needs to be plugged into the equation and it should equal zero. That x point is the solution.”

[I responded “ excellent explanation of the use of the x intercept graphing method of solving a linear equation in one variable….Is this the method you would use for solving 2x + 4 = 0?”]

2. **Compare and contrast the similarities and differences between solving a linear equation and a linear inequality.**

“A difference between the two [equations] is that with an inequality, the function is not set equal to another number, but to a value that is greater than or equal to the answer. A similarity is that you can solve both [equations] by adding, subtracting, dividing, and multiplying. The only difference is that with an inequality the sign (greater than or less than) changes when dividing or multiplying by a negative number.”

[The writer accurately identified a major distinction between the solving of an equation and inequality…. “dividing or multiplying by a negative number” causes the inequality sign to change direction. However, the writer refers to both inequality and equation as “equations” which is inaccurate and was pointed out. Algebra students often struggle with learning and using the correct terms and notation. This is an example where it was addressed before testing.]

Another student response is given below.

"Both can be solved symbolically, numerically, and graphically…When you solve its important to note only equations have equal signs, so only they can be set equal to zero. The solutions to inequalities can be found by setting two of them equal to each other, as opposed to 0."

[I responded "Your last sentence is unclear. I assume you were implying that you would use the graphing method to solve an inequality? One major difference between solving inequalities and solving equations (algebraically) is that when multiplying or dividing an inequality by a negative value, you must switch the direction of the inequality sign."]

3. **Discuss the relationship between the slope formula and the point-slope form of a linear equation.**

"The slope formula is used in algebra to find the slope of a line. Point slope formula is used to find the equation of a line when you have certain conditions. Ex. Slope = 2.4 passing through (4,5). The relation between the two could be best described by examples:

Slope formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Point slope $y_2 - y_1 = m\,(x_2 - x_1)$

And when you cross-multiply the point slope formula your result will be

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$. So you have a direct relationship between the slope form [*formula*] and point slope form of a linear equation."

[Again we see the struggle with the correct term to use "slope form" should be slope formula. However, this student clearly understands the development from the slope to the point slope form, although he did express it as a "direct relationship," which is a little unclear. Students also struggle with when it is okay to drop the subscripts and the idea that "x' and "y" could be substituted into the point slope formula for x_2 and y_2 without loss of generalization.]

4. **It has been said that piecewise-defined functions often serve as better models for real life situations than other continuous functions such as linear or quadratic functions. Explain why.**

"Piecewise-defined functions often serve as better models for real life situations than other continuous functions such as linear or quadratic functions because in real life situations it is possible that no single formula can conviently [*conveniently*] represent f. Because piecewise-defined functions typically use different formulas on various intervals of the domain, they are therefore better to serve as models for real life situations. Real life situations generally don't have [*follow*] a linear or quadratic function because people and earthly situations, usually not all things (x values) fall into the same category (equation.)"

[*Grammar and sentence structure aside, this was a super response! I wanted the students to see that math models for the real world are often not neat and tidy. FYI: The students who sat in the classroom lecture, as* *opposed to the online students who read their lesson online, did much better at responding to this question.]*

Another student response is given below.

"Piece-wise functions are best for real-life situations because it rarely happens that "f" is the same always. The book used the example of postage costs. If you mail a letter weighing an ounce or less you pay one price. If your piece of mail weighs over one ounce but less than two, you'll have to pay a different price, etc."

[I responded, "I agree real life functions are not always neat. Look at the greatest integer function on page ___for another look at postage costs."]

5. **The midpoint formula is used to find the midpoint between 2 points in the xy plane. How can this formula be adapted to find the midpoint between two points on a ruler?**

"The midpoint on a ruler can be found by solving the formula. You need to pick the two points you want to solve from the ruler. Solve the two points like you would any midpoint formula problem."

[I responded, "The midpoint formula is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$*. 2 points on a ruler would have only 2 values, not two ordered pairs (x,y).. Explain how you would adapt this formula."]*

The following examples of write questions also illustrate how student understanding of concepts was unveiled.

How does the discriminant tell us how many and what type of solutions a quadratic equation will have?

"The discriminant tells us how many and what type of solutions a quadratic equation will have by looking at its sign. If the discriminant is less than 0 there will be no solution, this is because you cannot take the square root of a negative number. If the discriminant is greater than 0 there will be two real solutions. And if the discriminant is equal to 0 there will be one real solution.

Discriminant → $b^2 - 4ac$"

[We had not discussed imaginary solutions yet in this course so this student's responses were good. Many students copied the rules from the text and I was unsure that they understood the concept of discriminant. Therefore, the day after I graded these responses, I gave a quiz to assess this knowledge. The students who copied the rules from the book scored poorly and those who wrote their own (as the student above did) did well.]

Not all write questions focused so explicitly on student understanding of content. Some examples and explanations follow.

Assess your progress in this course. How would you rate your understanding so far of the topics covered in class? What concepts have come the easiest and which have you struggled the most to understand?

[This is an authentic assessment question. The responses allowed me to personally address student struggles and to gauge the class' overall temperament.]

Was mathematics discovered or invented? Defend your answer.

[This write question was also given to a History of Mathematics class twice, first at the onset of the course and second at the end of the course. I found that the responses do not change much in terms of the invention or discovery theory. However, their latter defenses do contain more ammunition, mathematicians, dates, and anecdotes. The question allows students to clarify their thoughts on the subject as well as summarize what they have learned in the course.]

What famous mathematician has most influenced your life?

[Again this question was posed to a History of Mathematics course. I learned much about my students' early mathematical learning experiences from their responses to this question.]

Become an expert at a concept in this course! Pick a math concept and write everything you know about that concept. Pretend you are explaining it to another student who was absent the day we talked about it in class.

My earlier research on writing about mathematics had already demonstrated the following.

i. Writing promotes student comprehension of mathematics.

ii. Writing allows students to make personal meaning of mathematical concepts.

iii. Writing becomes a vehicle for dialogue between student and teacher.

iv. Writing allows for student reflection regarding the learning of mathematics.

v. Writing brings moments of clarity or genius to forefront.

vi. Writing permits authentic and alternative assessment for the instructor.

(Mower, pp. 310-320).

Moreover, the write question experience not only substantiated these findings but also intensified my commitment to asking students to write about mathematics.

Not all students embraced or enjoyed the write questions and some students began the semester by writing incomplete, vague, and clearly not well thought-out responses. However, in most classes, by the end of the semester, student responses had improved and communication skills were honed. I felt that if just one student moved outside his or her cube of limited mathematical understanding or just one student had an "aha" experience, then the experience was worth it to both the student writer and myself!

References

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Pat Mower is an assistant professor in the Department of Mathematics & Statistics at Washburn University in Topeka, Kansas.