An Action Research Study: Teaching a Mathematics Foundations Class through Discussion

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Abstract

Unless secondary mathematics majors see mathematics content courses taught in methods consistent with how they are later expected to teach, it is unlikely that future teachers will teach in a National Council of Teachers of Mathematics (NCTM) Standards approach. However, many mathematics professors are skeptical about how successful a mathematics content course taught in this nontraditional manner can be. This paper describes the results of teaching a foundations in mathematics course through discussion. Through this experience, the instructor has come to believe that a course of this nature not only can be taught through discussion, but some positive results can occur as a consequence.

There is often an inconsistency between how secondary mathematics majors are “taught to teach” and how they are taught in mathematics courses (Seymour, 2002). Secondary mathematics majors are most often taught to follow the principles and standards of the National Council of Teachers of Mathematics (NCTM, 2001) in their methods courses, but their mathematics content courses remain very traditional in pedagogy. Arguments can be made that teaching at a college level should be different from teaching at a secondary level, but it remains true that teachers teach as they have been taught, unless taught otherwise (Clark, 1997; DeLong and Winter, 2002; Ferrini-Mundy, 1998; Heaton, 2000; Leinwand and Burrill, 2001; Midddleton and Spanias, 2002). Students in mathematics education are told that a variety of methods exist to teach mathematics, but they rarely experience different methods in mathematics classes. The Mathematical Sciences Education Board (MSEB) has repeatedly called for "blending content and pedagogy" (MSEB, 1996, 2001) as have other national education committees, such as the Committee on Science and Mathematics Teacher Preparation (National Research Council, 2001b) and the

Mathematical Learning Study Committee (National Research Council, 2001a). There are undergraduate mathematics departments completely reforming their curriculum based on these issues (Pesonen and Malvela, 2000).

This issue led the author (a mathematics instructor, herein referred to as the instructor) to seek the help of a consultant in instructional development[1] and to decide to teach Foundations of Math and Geometry as a discussion course based on the readings in the textbook and supplemental readings. The instructor thought this would be interesting and challenging "as long as the chairs remained in straight rows," a sentiment born of teaching math traditionally for nine years. Discussion was chosen because the consultant and instructor believed that doing so would serve as a bridge to doing higher level mathematics; i.e., encourage the students' intellectual development (Sawada, et al. 2002). Moreover, the instructor hoped to provide a model for the students to take into their own classes someday. The particular course was chosen because it is required of all secondary mathematics majors and the topics in the course would seem to lend themselves nicely to this mode of teaching. Further, the small class size and number of meeting times per week facilitated discussion. The instructor searched for relevant pedagogical materials when preparing to teach the course. Resources can be found through the Mathematical Association of America (MAA) and academic journals (Boelkins and Ratliff, 2001; Hagelgans, et al. 1995; King, 2001; Meier and Rishel, 1998; Sterrett, 1992).

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[1] The instructional development consultant, Linda Rae Hilsen, was the author's colleague at the University of Minnesota Duluth. She contributed significantly to the success of the course in question as well as made editing comments on an earlier draft of this paper. However, before she could see this manuscript come to completion, she unexpectedly died. Her abilities as a teacher were profound. LeAne Rutherford, of the same department, stepped in and helped me finish this article. I am so very grateful for her insightful comments. Janelle Wilson and Jill Jenson, both of UMD, and an anonymous reviewer also made this a better paper.

This article is based on the instructor's experiences of teaching a mathematics content course using a discussion pedagogy and working with a consultant, and is offered in the hopes that other higher education instructors can entertain the idea of incorporating discussion in such classes.

Purposes and Goals

The instructor wanted to provide experience in a math content course taught with "reform" methods. It is not enough, however, to provide a course taught with nontraditional methods, but to do so in such a manner that the students rate the course positively. In addition, the instructor wanted to accomplish specific goals, and then evaluate how well the goals were met through the discussion format. The goals were to

- move students up the ladder of Perry's developmental scheme[2]
- lay the foundation of and the bridge to higher order mathematics.

The Setting

The course was taught at a comprehensive regional university with 11 bachelor's degrees in 70 majors, a two-year program at the School of Medicine, and graduate programs in 18 different fields. The university views itself as an alternative to both large research universities and small liberal arts colleges. It is a medium-sized campus of a major university.

One of the courses that is required of secondary mathematics majors is a content course entitled "Foundations of Mathematics and Geometry." The course catalog reads:

[2] Perry's Scheme will be described later in the paper.

> Math 3110. Foundations of Mathematics and Geometry. Introduction to foundations of mathematics. Non-Euclidean geometries, postulational systems, and models. History of mathematics. Importance and use of mathematics in modern society.

It is a 5-credit semester course meeting for 50 minutes Monday through Friday for 15 weeks. Twenty-two students were enrolled in the course during the semester in question.

Getting Started

Generally, discussions of textbook chapters, as well as discussion of outside readings and assignments, formed the core of class activities. Before the instructor learned the students' names, discussions and responses were heavily centered in the first two rows of the classroom, while students in the back remained silent, even though the instructor had eye contact with all students. Even though many were listening and not actively participating, they did not appear to be opposed to the teaching method and when directly asked by the instructor, they responded that they were content with the manner in which the course was proceeding. However, during the third week, the instructor asked the students to put in writing how the course was going, and some students responded that they did not feel they had the opportunity to participate, because some students were monopolizing the discussions. These responses from students concerned the instructor.

After all, to verbalize information helps students retain it (Finkel & Monk, 1983; Glassman, 1980; Myers, 2000; Weisz, 1990; Welty 1989). The consultant suggested that if the instructor knew the students' names, the instructor would have the ability to control the source of responses and, perhaps, all students would then participate. To learn the students’ names, the instructor asked the students to bring in pictures of themselves. These were photocopied and hung in the instructor’s office, enabling the instructor to quickly learn their names. Once the instructor knew students’ names, she could directly ask a student if she or he agreed or disagreed with the points being made. Students responded well, and the instructor could engage a student by calling on one and continuing to ask probing questions. Very quickly, the dynamics in the classroom changed, and soon all students were contributing to discussions. Student learning was enhanced as a result of this participation as will be apparent in the next three sections of this paper, which contain a discussion of meeting of the goals, as well as students' reaction through the students' evaluations of the course.

Meeting Goal One

The first goal was to move students up the ladder of Perry’s developmental scheme. Two examples are offered to show that this goal was met. The first concerns proof or justification schemes, and the second concerns students' responses to a question "What is mathematics?"

Proof or Justification Schemes

One topic in the course was the development of proof or justification schemes. The instructor used the categories given by Sowder and Harel (1998): externally based proof schemes (authoritarian, ritual, and symbolic), empirical proof schemes (perceptual and examples-based), and analytic proof schemes (transformational and axiomatic). At one point, the instructor told students that most of their secondary students would operate on the basis of empirical proof schemes. As the class discussed the proof schemes, Perry's developmental theory was also discussed (Perry, 1970). It may seem that one has nothing to do with the other, but the reader is promised a connection.

In Perry's development theory, there are nine positions that college students progress through in their intellectual development. The three broad positions of dualism, relativism, and commitments (each of these three has three substages) are sufficient for the illustration given here. In the dualism stage, students believe that every situation is an either-or, with only one side being correct. In relativism, students believe that there are numerous possibilities and it is not possible to declare one correct. Students in the stage of relativism have no commitments to any one of the possibilities. Once students move to the commitment stage, however, the students do commit to a particular position. Of course, not all students reach this stage.

Teaching mathematics in the discussion format facilitates movement through Perry’s stages because as students develop, mathematics is seen as more gray than black and white. See Figure 1. Moving down the columns in this figure implies increasing developmental growth and increasing critical thinking. Taking the softer view of math might allow students to move in their development from dualist to relativist. In the beginning of the course, many of the students wanted to “do” mathematics as opposed to understanding its underpinnings. Rishel (2000) describes the implications of the Perry model for mathematics by commenting that “sometimes students say that they like mathematics ‘because in math all the answers are known’” (Rishel, p. 88). Further he states the following.

> Thus, their subsequent problems with ability to prove theorems can often be traced to their view of education itself, and not simply in their refusal to get

> the “theorem-proof” concept. One possible implication of this last conclusion, if true, may be that courses in proof theory for sophomores must be constructed to take into account the question of students’ belief structures. (Rishel, p. 88)

Perry	Math Method	Bloom
Dualism	• "do" math • one answer • no need for proof	Knowledge
Relativist	• wants to understand math • able to discuss math • sees a need for proof	Analysis Comprehension
Committed	• teach in ways that match commitment	Application Synthesis Evaluation

Figure 1: Relationship Matrix

In this sense, the students in this study were operating from almost the same developmental position as their future students. Many secondary students operate with a sense that mathematics has exactly one answer and that a set of examples proves something. Here is the promised connection. Secondary students operate from the externally based proof schemes or at best the empirical proof schemes. This often bewilders secondary teachers. Yet, secondary mathematics majors operate from a similar position. It is more difficult to recognize this position under the Sowder and Harel system, but consider the Perry developmental ladder and one can recognize it easily. Granted, Perry was not describing a proof scheme, but his scheme works for this illustration. And actually, in one sense, the instructor was also operating from a lower position.

To demonstrate this last similarity, moving the chairs in the classroom facilitates discussion, helps the instructor break the habit of lecturing, and creates a much more student-centered classroom. In this study, the instructor was afraid to move the chairs. She believed that if other university mathematics instructors walked by the classroom, they would wonder why she was not teaching the mathematics class. She did not want the students to move the chairs for fear that the course would not be taken as a serious mathematics class by either the students or other professors. This was not the case. An evaluation was given after a month of the class, to see how the class was proceeding for students. Students remarked on these evaluations that they needed to move the chairs to fully engage in discussion. Although the placement of the chairs might strike the reader as trivial, the students did show evidence of growth in their concept of what it means to do mathematics. To do mathematics began to include discussion of mathematics, and not just working problems. As one student wrote, "It is important to recognize that as one's study of mathematics deepens, the definition is subject to change."

Further evidence of student growth occurred directly in their concepts of proof. Since these students were juniors majoring in secondary mathematics, they already had moved beyond the idea that a number of examples (no matter how many) do not prove a mathematical statement. Preponderance of the evidence is not sufficient in mathematics for proof. However, they showed evidence in their writing of understanding that this is a developmental process (e.g., "…from applying small truths over and over again to find bigger truths."). Further, they showed evidence of seeking the proofs for themselves at an axiomatic level. The instructor had taught this course before, without the discussion format. Although no claim is made to having a controlled study, students taught in the traditional manner did not move into the level of *needing* proof. Students may well have been *able* to provide proofs, but they did not express a *need* to provide proofs.

The instructor moved out of her notion that chairs in a mathematics class must be in rows. The students moved out of their notion that discussion does not belong in a mathematics class. Everyone's sense of mathematics and/or mathematics teaching moved up from a dualistic position in the Perry scheme to a relativistic one. Perhaps most importantly, students moved into a deeper need to see proofs in a mathematics class.

Response to "What is mathematics?"

As mentioned previously, students were asked the thought question: “What is mathematics?” The same question was asked periodically so as to see how students’ definitions changed over time. This section examines students’ responses to the question in detail to see growth or developmental movement as well as to gain evidence of how much students really took from the course and how much they were able to claim as part of their own knowledge base.

"What is mathematics?" was asked at three times in the semester course, at the beginning, middle and end. See Table 1.

Response	Everything	Order	Problem solving	Way of thinking	Study of numbers	Symbols	Game	Logic
Beginning	40%	20%	10%	10%	8%	6%	0%	6%
Middle	30%	35%	10%	5%	10%	0%	5%	5%
End	20%	30%	5%	25%	5%	5%	5%	5%

Table 1: What is mathematics?

Response one. The question was first asked four days into the course. Approximately 40% of the students said some version of "mathematics is everything." Examples of those responses are:

- What is mathematics? Well, this is a pretty difficult question to answer. I think mathematics is present in everything. In everything we do, in everything we see, in everything around us.
- Mathematics is everything. Everyday we see cars being driven on roads, boats floating in the harbor, or airplanes flying in the sky. All of these things, and so much more, are a product of mathematics.
- Mathematics is everything. Without it nothing would exist.
- Mathematics isn't just one thing. Mathematics is everything. It is an art, a science, numbers, problem-solving, equations, graphs, & so on.
- I believe mathematics is everything around us. We use math as a tool and a symbol to reach an understanding of something.
- Mathematics is more than the science or study of anything. I can not put my finger on one thing that mathematics is. It is the underlying force that has allowed our race to achieve amazing feats.

Approximately 20% of the students said that mathematics is a method of ordering the world. Examples of those responses include:

- Mathematics is the science that uses rational reasoning to come to a conclusion that is deemed logical by the system and rules set by the science.
- Mathematics is an attempt to understand the world around us.
- Math was becoming a way of producing order and connecting to the real world.
- Mathematics is a way to study and understand the world.
- Mathematics is the use of numbers and variables to figure out why things are the way they are and also to figure out how we can achieve other things.

Approximately 10% of the students said that mathematics is problem solving. Consider the following examples of those responses:

- I think that problem solving is a large part of mathematics.

- The main use of mathematics is to solve problems and make things clear.

Another 10% called mathematics a way of thinking. For example:

- Mathematics is an intimidating, complex way of thinking.

The remaining students called mathematics the study of numbers, symbols or logic.

- Mathematics is the study of numbers.
- I believe that mathematics is the study of numbers and values, and their correlations.
- Mathematics is a system of rules and symbols used to define the universe and how it works.
- Mathematics is to me a system of logic that can be applied to every day situations.

Response two. The second set of mathematical definitions was collected at the midpoint of the semester course. Students' ideas had changed quite a bit. Fewer students said that mathematics was everything. However, more students said that mathematics placed "order" on everything. Students were switching from a point of view that mathematics was a set of content (the study of things, symbols, or numbers) to mathematics was a process (something that one does, manipulates, or thinks about). Mathematics was being viewed as more dynamic and as science to be applied to the world. Examples of students' comments follow:

- Math is the tool we use to help understand and describe things around us.
- Mathematics is based on reasoning. It is the science of patterns and it consists of conceptual ideas.
- Mathematics has moved forward to being more of a reasoning force.
- Mathematics is what allows the universe to operate with such order.
- Mathematics is what enables humans to make sense of creation.
- Mathematics is the source of order in life.
- Math is huge.
- Math is an art, science, it's logic, music, numbers, problem solving, time, length, speed, distance and so on.
- Mathematics is the study of quantities.
- Mathematics is the study of correlations of values used as a tool to describe things and forces around us.
- Mathematics is a very valuable game.

Response three. The third response was collected at the time of the final exam. Students continued to change in their responses. Again, fewer students said that mathematics was everything. More students thought of mathematics as a way of thinking. Most significantly, many students commented on how many different ways they were now viewing mathematics. Paradoxically, when students decreased their descriptions of math "as everything," they really were increasing and broadening their view of math. In reality, they were thinking of many different views of mathematics and were more able to be more specific about it. Examples of students' comments are listed:

- At the end of the course, I'm more willing to give math a much broader definition. I am seeing math in more areas of life.
- Mathematics is a realm of thinking that we have created.
- Mathematics is a thought process based on logic and set rules.
- Mathematics is a combination of three things: reasoning, the science of patterns, and considering math as a tool.
- Mathematics is an attempt to make sense of the world. Creation has inspired man to discover or create mathematics. It has then become more abstract as the passion for the subject becomes the primary focus, rather than its applicability.
- Mathematics is a system of rules and symbols used to describe and define the universe.
- Mathematics is a very important addictive game.
- I believe that math is everything in the sense that it can be applied to everything.
- Mathematics is many different systems of numerals that make it easier to solve different problems.
- Mathematics is the science of following logical rules to skillfully manipulate numbers and symbols for the purpose of solving problems.

The qualitative evidence that students provided and was analyzed in Table 1 demonstrates fresh flexibility in conceiving of both content (math) and process (teaching). Change has occurred. Of course, the instructor and consultant believe that this change was directly influenced by the reform method of teaching. Much of the research literature on reform teaching results will support this view (see Senk and Thompson, 2003).

Meeting Goal Two

The second goal was to lay the foundation of and the bridge to higher order mathematics. Assessment results are provided to show this goal was meet. Assessment is a concern in math courses. This section discusses how assessment was incorporated into this class and then shows results of students' growth on some of the assessment items.

The breakdown for course grades was a midterm (20%), a final (20%), a class presentation (20%), and reading and writing assignments (40%). The reading and written assignments included such things as keeping learning logs, outlining chapters, responding to thought questions, solving problems, and writing exam questions. In the learning logs, students wrote short statements about the main points in a reading or discussion. The thought questions required a few paragraphs responding to such questions as:

- What mathematics should be taught?
- How should mathematics be taught?
- What are the goals of mathematics education?
- Who should take mathematics?

The purpose of the majority of these assignments was to create an anticipatory set for the students so they could participate in the discussion in class. The reader is referred to Angelo and Cross (1993) for an excellent source of assessment ideas.

The assignment of writing exam questions was effective in forcing students to think deeply about the content being covered. Students were asked to write a question appropriate for a test and answer it from any of a given set of material covered in the course. Students first wrote questions that asked for the “naming” or “listing” of things, memory questions which require the lowest level of cognitive development of thought processes; this utilizes the lowest level (knowledge) in Bloom’s taxonomy (Bloom, Englehart, Furst, Hill, & Krathwohl, 1956), the stages of which are knowledge, comprehension, application, analysis, synthesis, and evaluation. Over time, students began to ask and answer more thoughtful, deeper questions further along Bloom's taxonomy. Here are some examples of students’ test items and a suggested level in Bloom’s taxonomy:

- What is the difference between inductive and deductive reasoning? (analysis)
- Consider one of the contributions made by the Greeks and explain how it changed the study of mathematics at that time. (knowledge, analysis, and synthesis)
- Describe in your own words the difference between infinite and potentially infinite. Why do you think there is so much division among mathematicians on the subject? What view do you take? (comprehension, synthesis, and evaluation)

Surely higher level thinking and much more than rote memorization was needed to write these items and would be needed to successfully answer them. The students moved up Bloom's taxonomy through both the questions they posed and the answers they gave.

The questions created by students were supplemented with instructor's items for the midterm and final. In this course, expectations had been clarified from the beginning. Students had had practice in class answering many of the same types of questions used on the examinations. There were no surprises for the students. They had been taught how to take such tests. It is surprising how few college teachers do this, which is why many students have justifiable complaints about how their teachers test. One wonders if college teachers fear that it is the same as teaching toward a test, when it is simply allowing students to practice on similar types of questions. Effective teachers give their students tests which reflect what they have been grappling with in class (NCTM, 1995).

For the class presentations, students were asked to present more details about a mathematical topic that was covered in the course. The requirements were instructor approval of student projects; length of at least 15 minutes; and some type of handout. The presentations were both peer- and instructor-reviewed. The criteria were clarity, level of interest, and mathematical correctness.

Assessment Results

The scores on the midterm and final reflect that students did learn the content knowledge. The mean on the midterm was 88 (standard deviation of 10.01), and the mean on the final was 89 (standard deviation of 7.76). (Note that when the instructor taught this course without discussions, the final mean was 62 with a standard deviation of 9.03. Again, this does not constitute a controlled study.) It was not necessarily true that those who did the best job on the writing assignments did the best job on the tests. However, all test scores were relatively high. For example, on the final, the lowest score was a 74; the median a 92; the mode a 83.

Student Reactions

The responses students gave to evaluation opportunities are given next. This is not intended to demonstrate student growth, but to give some evidence that the course was successful in students' eyes. If the students' opinion of this course was negative, then it is difficult to say if the purpose of providing a content mathematics course taught in reform manners has truly been fulfilled. However, the evaluations were indeed positive. Two types of course evaluations were administered: Teaching Analysis by Students (revised from Erickson, 1966) and the standard form offered by the university and used by the Mathematics Department.

Teaching Analysis by Students

The formative and summative evaluation instrument "Teaching Analysis by Students" was given twice during the semester. This form consists of 25 teaching behaviors which are ranked by the students on a scale of one to four (one being the perfect score). Space constraints prevent a complete reporting of the results. The items with the three highest and lowest means are given in Table 2 with the standard deviation given after the mean. As can be seen from the means, even the bottom scores were quite high. Students were satisfied with the course and the instructor's performance in the course.

Equally informative were students' responses to three questions of interest. What is the instructor doing that helps you learn? What is the instructor doing which gets in the way of your learning? What suggestions do you have for changing the course? Following are a subset of these responses after the fifth week.

Midpoint in the semester	Note: 1 is high on this scale. 1.00 is a perfect score. The instructor's performance in …	Mean	SD
Top 3	• responding to questions raised by students • asking thought-provoking questions • displaying enthusiasm in teaching the subject	1.00 1.21 1.21	0.00 0.42 0.42
Bottom 3	• assigning useful reading and homework • using a variety of teaching techniques • keeping me informed about how well I am doing	1.74 1.79 1.83	0.65 0.71 0.71

Final Week	The instructor's performance in …	Mean	SD
Top 3	• explaining what is expected from each student • displaying enthusiasm in teaching the subject • inspiring excitement of interest in the content of the course	1.00 1.00 1.06	0.00 0.00 0.25
Bottom 3	• making effective use of class time • keeping me informed about how well I am doing • taking appropriate action if students appear to be bored	1.44 1.50 1.50	0.51 0.63 0.52

Table 2: Evaluations

What is the instructor doing that helps you learn?

- Incorporating direct relations to the secondary level to providing additional reading available in texts, magazines and journals. Keeps discussion open and leaves plenty of room for debate.
- Being very receptive, knowing a lot about examples or handouts in class.
- Asks thought-provoking questions to think about.
- A huge reason that helps me learn is the atmosphere of the classroom. Everyone feels comfortable.
- I like how we read a chapter on our own and then in class pick it apart through discussions.
- The written assignments help me to connect with the material.

What is the instructor doing which gets in the way of your learning?

- Gives attention to louder students instead of students who are quiet and reserved. Should try to get other students voices to be heard.
- Sometimes I think there is too much information to know.
- Sometimes the readings can get too long.

What suggestions do you have for changing the course?

- If there were anyway to incorporate more math, that would be good because that is what people in the class are interested in and it seems to be like a history class.
- More discussion in groups—maybe more with moving desks around.

Although some students realize that the class benefits from the discussion, others are mired wondering where is the math? It is difficult to convince students and fellow faculty that the math is not centered in the symbolic placement of the chairs. Of course this is not difficult to understand because even the instructor did not want to move the chairs for this very reason.

When these same questions were asked at the end of the semester, the students did not offer as many comments as on the first round. A subset of the responses does follow, however.

What is the instructor doing that helps you learn?

- Presentation style
- Excellent all around teaching methods!!
- I liked how Carmen didn’t lecture the full time, she created an environment which allowed students to speak and discuss things when they wanted to.

What is the instructor doing which gets in the way of your learning?

- At times she is distracted by students and gets off track.
- Every once in a while, I have a hard time deciphering what might be important, but most of the time it’s very clear. That is all—nothing else.
- Sometimes assumes people understand concepts when they may be new to some people.

What suggestions do you have for changing the course?

- A more specific syllabus.
- Make up a more solid collection of course goals so students know right away what’s ahead. Perhaps since so much of it is discussion, try to get the class in a room that encourages interaction.

Students were much more inclined in the second set of evaluations to view the discussion part of the course as important, and, in fact, many of the suggestions revolved around facilitating that aspect of the course.

University Evaluation Instrument

On the final day of class, the regular evaluation forms required by the Mathematics Department were administered. This form consisted of 26 questions that students respond to on a scale of 1 to 6 (1 is most strongly disagree, 2 is strongly disagree, 3 is disagree, 4 is agree, 5 is strongly agree, and 6 is most strongly agree). The lowest score out of the 26 questions was a 5. Some selected responses follow with mean and standard deviation.

- Overall, the instructor presented the subject effectively. (5.67, 0.48)
- The course as a whole was good. (5.52, 0.68)
- Overall, I learned a lot in this course. (5.43, 0.81)

Of course, the instructor and the consultant were pleased with the evaluations. The results were consistent between the evaluation forms, and students were overall satisfied with the course.

Purpose and Goals Revisited

From the beginning of the course, the purpose was to provide experience in a math content course taught with a reform manner. The students experienced a different type of mathematics class and had positive opinions about the experience. Through the semester the students came to value the methods. Perhaps this will make students more willing to try reform methods when they are teaching mathematics. Other evidence (such as the students' responses to "What is mathematics?") shows that the students grew in their appreciation of mathematics, and learned to communicate about mathematical issues.

Trying reform methods for the sake of trying reform methods was not the purpose, nor should it be for students when they go out to teach. Students should grow mathematically. The instructor wanted to lay the foundation of and the bridge to higher order mathematics. In discussing the course after its completion with a group of colleagues, one mathematics professor remarked that she taught a mathematics class immediately following, in the same room as the class in this study, and shared some of the same students. (Her course would be considered by the department as an upper level course requiring more mathematical ability than the one described in this article.) She went on to remark that she could tell which students were shared because they were the students who were able to communicate mathematically, contribute to the course, and learn the material. While anecdotal evidence, the instructor and consultant could think of no stronger endorsement.

Previously in this paper, the results of more traditional methods of endorsement were given, i.e., test scores. As stated then, these test scores were considerably higher than when the course was taught under traditional manners. In fact, everyone passed the course in question, and previously many students had to retake the course. This is not to suggest the course was easier this time, as many of the test questions were the same. The students grew in their mathematical skills.

Another goal was to move students up the ladder of Perry's developmental scheme. One student wrote the following on his last definition of mathematics:

> The average person probably would say that mathematics is arithmetic, algebra, numbers, or something along that line. It is hard to describe this word that we all love and see as beautiful. Mathematics has a certain beauty to it, but the average person cannot see that beauty because you have to have a deep understanding and love of mathematics in order to see it. This beauty is what makes mathematics hard to describe. My definition has changed three times, and if I were asked again I am sure it would change.

Many of the students moved from thinking that it is unnecessary to ask "What is mathematics?" (because it is so obvious what the answer is) to thinking it is an unanswerable question, a question with many answers, or a question that depends on the context or situation for its answer. This is definitely movement through Perry's stages. Movement through the Perry stages was a general trend. It occurred not only in the question of "What is mathematics" but also in the students' views of proof, and in fact in their understanding of the content of the course (as is evidenced in their written test questions).

Final Thoughts

Mathematics content courses have not often been taught through discussion (MSEB, 1996, 2001). However, not only can mathematics content courses be taught through discussion, but at least some of them should be (National Research Council, 2001a, 2001b). Of course, balance is needed, and it would be foolish to advocate teaching all mathematics courses as discussion courses. However, departments might consider designating certain courses as discussion courses, and a foundations course is a rather appropriate choice. It is often required of secondary mathematics majors, who may themselves teach, and these are the very majors who would most benefit. They need to experience this method and sense its potential.

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