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African Fractals and Ethnomathematics

Heather Allentuck, Taylor Davis, Joshua Roach
Miami University

Abstract:
Traditionally, classrooms focus on Eurocentric mathematics; however, math is evident all over the world. In this paper we focus on the use of fractals in African culture, an element of ethnomathematics. Through studying Ron Eglash's book African Fractals: Modern Computing and Indigenous Design we analyzed how different architecture in the Kotoko, Ba-ila, and Nankani peoples uphold the five essential elements of fractals – self similarity, recursion, infinity, scaling, and fractal dimensions – as defined by Benoit Mandelbrot. This article delves into the largely unstudied realm of African Fractals in relation to ethnomathematics and provides practical classroom application, exposing students to culturally diverse mathematics.

Introduction

Mathematics is evident all over the world in many different cultures and societies; however, American classrooms tend to focus on Eurocentric mathematics. Through an examination of African fractals in this paper, we explore the realm of ethnomathematics, defined as "the mathematics... practiced among identifiable cultural groups" (D'Ambrosio 1985, p. 45). Fractals are fascinating, repeated patterns that are found in art and religion throughout the continent, but this article concentrates on the use of fractals in architecture and their application in mathematics classrooms. Through the exploration of these figures, students can learn about scaling shapes, self-similarity, and Euclidean geometry, which apply directly to the *Geometry and Number Systems Common Core Standards* for the middle grades (CCSSM, 2010). Additionally, we highlight web applets and other resources that assist in the teaching of fractals.

Fractals

The past fifty years have seen a rapid increase in fractal research in fields such as geology, anthropology, archaeology, and mathematics (Brown, 2007). The implications of fractals and their occurrence in nature and human culture continue to intrigue scientists and mathematicians, but much research remains. According to Benoit Mandelbrot, the father of modern fractals, a fractal is, "a set with self-similar geometry and fractional dimensions” (Brown, 2007, p.40). Below in Table 1, we explain the five main components of fractals.

Self-similarity	A pattern that is exactly the same regardless of the scale (Brown, 2007)
Recursion	The process of repeating the same design, which forms a fractal pattern (Roy, 2003)
Scaling	A change in the linear dimensions or size of the shape by a constant amount (Roy, 2003)
Infinite	Fractal patterns are repeatedly indefinitely
Fractional Dimensions	As you measure the outline of an object with increasingly smaller units of measurement, the detail of the outline will increase.

Table 1: Fractal Components

African Fractals

In the mid 1970’s, Ron Eglash, a professor from The Ohio State University, was flying over African villages taking aerial photos and noticed that some village layouts followed fractal structures. He brought this observation to other experts in the field of geometry leading to more examples of this phenomenon being discovered across Africa.

<u>Kotoko Culture</u>

One of the villages Eglash photographed was Logone-Birni, in Northern Cameroon, built by the Kotoko people. The city was created through a process known as “architecture by accretion” (Eglash, 1999, p. 24), meaning that one structure was built, and then others were connected to it. Figure 1 shows both an aerial view of the village and a computer illustration of its structure. The illustration shows how the original rectangular building, rectangle 1, was created then the additional iterations were later added.

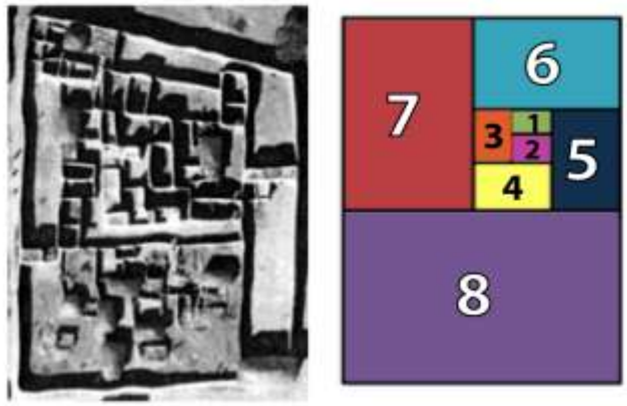

Figure 1: Aerial view and computer model of Logone-Birni (Eglash, 1999, p.21)

The recursion and self-similarity of the Logone-Birni's village structure is displayed in the computerized model, Figure 1. Rectangle 2 is self-similar to rectangle 1 because their scales are identical. By using rectangle 1 as a template for each iteration, the process of recursion creates rectangle 2 and each of the following boxes. This self-similar, recursive pattern created the city of Logone-Birni.

Additionally, the Logone-Birni village shows the infinite property of fractals since the rectangles continue to iterate, with each iteration making the pattern larger. Since the pattern can iterate an unlimited number of times, the pattern can become infinitely larger. Some African cultures depicted infinity by drawing a snake in a circle, trying to bite its own tail, as shown in Figure 2. The image demonstrates this concept because a circle continues indefinitely without any breaks (Roy, 2007).

Figure 2: African Infinity Snake (Roy, 2003)

<u>Ba-ila Culture</u>

Fractal architecture is also found in the Ba-ila culture as seen in Figure 3. This culture is relatively isolated from other African tribes, meaning that fractal structures emerged almost independently of outside influences (Eglash, 1999). As seen in Figure 3, the village is built in an elongated ring “seed” shape, within which each family lives in a compound similar to the larger structure of the village. The village shows the concept of scaling because the compound’s shape is scaled down into smaller versions to create the family homes. Each new iteration is the same distance proportionately away from and sized to the previous iterations.

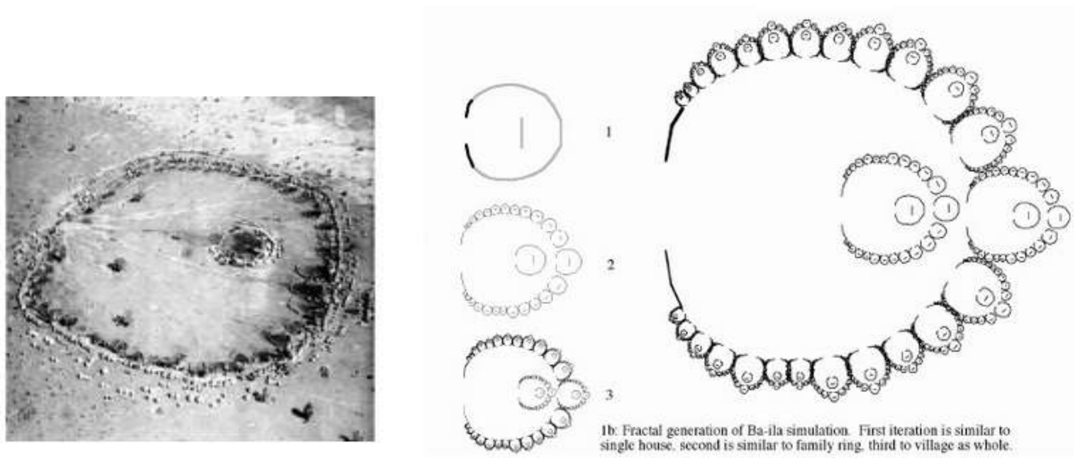

Figure 3: Aerial view and computer simulation of Ba-ila Settlements (Eglash & Odumosu, 2005)

<u>Nankani Culture</u>

The homes created by the Nankani culture use scaling circles to represent their social structure, representing the passage from birth to death. A child is born in the birthing room and then is said to crawl to the courtyard into the village and then to the world, moving through the various circles (Eglash, 1999). The circular arrangements, in Figure 4, demonstrate fractional dimensions. If the circles were measured with a large unit of measurement like a meter, then it would not be possible to measure all of the nooks and crannies in the house structure. However, if the Nankani home is measured with smaller units of measurement like a centimeter, then it can more accurately define the nooks and crannies leading to a more detailed outline. Essentially, fractional dimension consists of the clarity in detail of the pattern being examined due to the size and number of the units outlining the pattern.

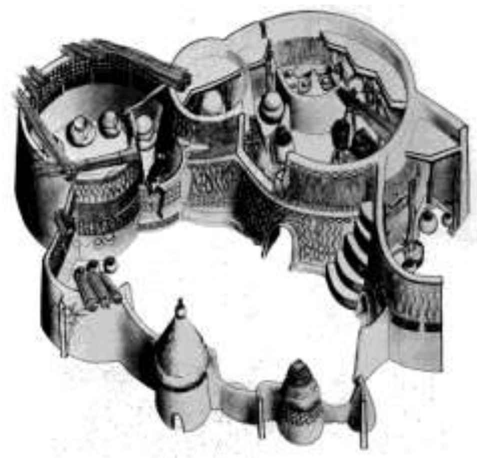

Figure 4: Artist Rendering of a Nakini house (Roy, 2003)

<u>Creation</u>

Fractals are not the simple result of unintentional creation or imitation, but instead a deeper reflection of the cultures that created them. For example, the fractal “accretion architecture” described from the Kotoko people is a physical representation of their social structure. The Kotoko tribe has a “bottom-up” social structure consisting of smaller, decentralized social groups with little political hierarchy (Eglash, 1999, p. 21). This structure relates to fractals because each family in the tribe builds their home as an iteration of the village’s design according to their social status.

Classroom Applications

Discovering fractals, especially through online resources, can help students “explore mathematics by making spatial connections in a way that is almost tangible” (Milner, Hodgson, Moore, & Wheatley, 2002). Fractals can also be used to “recognize and apply geometric ideas and relationships in areas outside the mathematics classroom” (NCTM Geometry grades 6-8, 9-12). Using the NCTM Standards, African fractals can be used not only in mathematics, but also to teach many scientific concepts such as systems, order, and organization. Likewise, fractals can be used to integrate world architecture, history, and culture into the mathematics classroom.

Online Resources

Currently there is a dearth of online applets and other resources to teach African Fractals. However there are numerous online resources that can be used to help students visualize fractals and their structures. For example, applets like the one pictured in Figure 5 allow students to design their own fractals and explore the concepts of iteration and recursion. Similarly, various computer programs like Microsoft Word and Excel can be used to create fractal designs with the Auto-Shape feature (Milner et. al, 2002). Another good resource for teachers is the Fractal Foundation (http://fractalfoundation.org/), which has sample lesson plans and other activities for implementing fractals in the classroom.

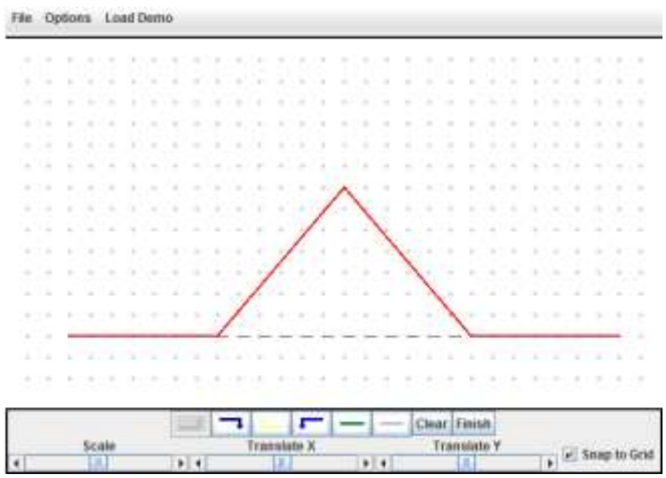

Figure 5: Iteration and Recursion Applet (Roy, 2003)

Summary

The field of ethnomathematics is growing every year and offers great opportunities for teachers. By incorporating African fractals into the classroom, students have the opportunity to master important mathematical concepts while at the same time validating non-Western mathematics. African fractals illustrate the main components of fractals: self-similarity, recursion, infinity, scaling, and fractal dimensions as evidenced in architecture of the Kotoko, Ba-ila, and Nankani peoples. Altogether, African fractals can provide a valuable addition to your classroom.

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Active Learning in Mathematics

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Abstract:
Active Learning strategies have been demonstrated to dramatically improve student performance and retention in the Science, Technology, Engineering and Mathematics (STEM) fields. In this study, a group work active learning strategy was applied to a group of Introductory Algebra (Math 093) courses at Baton Rouge Community College. The performance of the students in these classes was compared and contrasted with their performance in the course under standard lecture based learning as well as other students in the same course used as a baseline.

Introduction

There has been push towards active learning techniques in the classroom in an attempt to increase student retention and performance. In 1991, Bonwell and Eison introduced the term *Active Learning* to the educational masses, referring to a strategy of engagement in the classroom involving activities rather than lecture (Bonwell and Eison, 1991). The notion was simple: engaged learners will retain more information longer, and hence perform better, than non-engaged students. Since then, many studies have been done to demonstrate the efficacy of active learning techniques. In 1998, Richard Hake (Hake, 1998) conducted a famous study involving introductory physics classes, demonstrating a significant improvement in student conceptual knowledge in active learning vs. lecture based environments. These results are very promising for the STEM field, but a cursory look at the data available indicates that most of the focus has been on science and engineering courses, not pure mathematics.

Methods

In Fall 2012, I performed an experiment in the four Math 093 courses I taught. Math 093 is an introductory algebra course that begins with basic manipulation of linear equations and covers material through solving quadratic equations by factoring. The purpose was to gather data on the effectiveness of active learning strategies in the classroom. Each of these classes initially contained from 24-30 students.

Two of these classes were part of a grant program called Title III, and hence had a student worker assigned to help in the classroom. The student worker did not instruct, but was very useful during the group work time as an extra pair of hands.

Each of these classes was run in the same format. The first half of the course, through midterm, was done in the classic lecture technique. After midterm, the teaching methodology was changed to incorporate active learning. Each class was divided into groups of two or four (depending on the remaining number of students in the course). A typical class would consist of a very brief example outlining the basic problem techniques to be covered that day. Then, the class would break apart into groups to perform several problems (typically 3) using the new technique. It is important to note that examples involving classic pitfalls inherent in some of these techniques were left to the students to discover in groups, not in the lecture problem.

> *As an example: When solving linear equations, a common mistake for students is to neglect the -1 coefficient on the outside of a set of parentheses. This typically results in the student failing to distribute the -1, and hence arriving at an incorrect answer. An example involving this pitfall would intentionally be left for the group work time, allowing the students to discover the trap, and hopefully better retain the information*
> .

Each of the groups was determined through a seeding process based on their midterm grades. The student with the best test score was paired with the worst, second best with second worst, etc. The purpose here was to attempt to ensure that two very poor students were not paired with one another. No other parameters were used in group assignments.

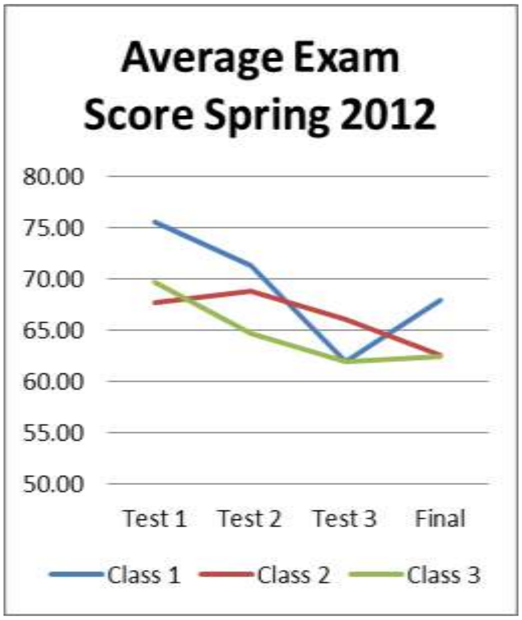
During group work time, I would walk around the room briefly visiting each group. The first reason for this is to answer potential questions each group may have about the current assignment and to suggest some of the internal questions the students should be asking themselves as they work the problems. The second is to check solutions so that each group does not waste valuable time unsure who has the correct answer (if anyone). The third is to refocus some groups that are potentially getting off task.

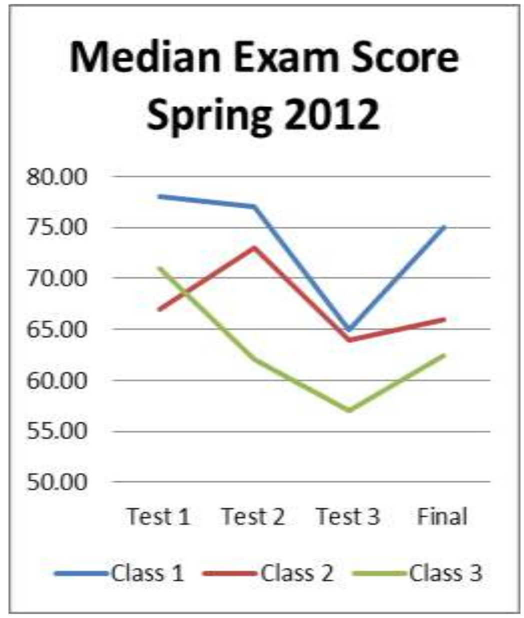
The data given below represents the test scores from each of the four classes, as well as the averages on each test for each class. For the purposes of this study, any student who did not complete at least the third exam will be removed from the data set. Subsequently, the changes between Test1-Test2 and Test2-Test3 were calculated for each student who completed the third test. Additionally, the data were reconfigured to represent only those students who completed the course (took the final exam). These data were calculated to represent the change Test1-Test2, Test2-Test3, Test3-Final, as well as the change in the average of Test 1 and 2 to the average of Test 3 and 4. All of these changes are reported as percent increases or decreases.

Included also are the same data obtained from the Spring 2012 semester, in which active learning techniques were not used. These data are to serve as a baseline for our interpretations. The first three exams given to the classes were identical. Final exams were not identical, but were sufficiently close such that the bias introduced would be minimal.

Data

Data shows that the baseline performance for 093 students shows a dip in the average scores from test 2 to test 3 (7.14% decrease) over that of test 1 to test 2 (5.40% decrease). Among the students who completed the entire course, there was a small increase in performance on the final exam, what amounted to an increase of 1.81% (though the explanation for this is due in part to the fact that the regular semester exams and the final exam were written by two different people). Overall, the difference in performance from the first two exams to the last two exams amounted to an 8.25% decrease in average scores.





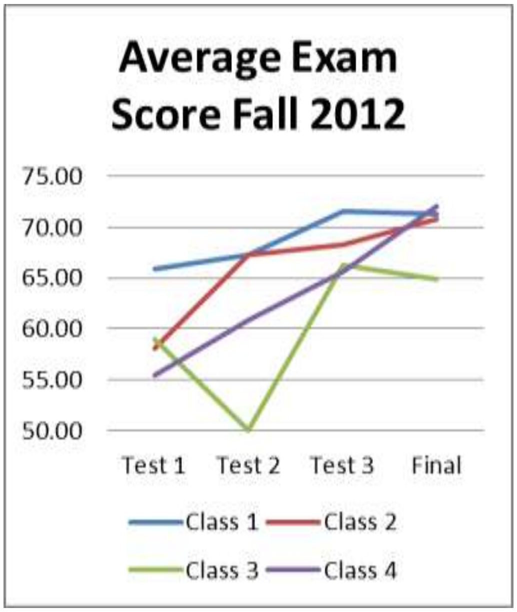
AVERAGE

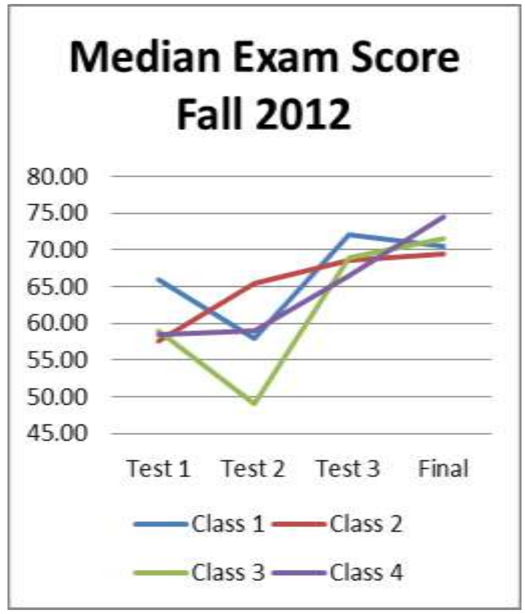
	Test 1	Test 2	Test 3	Final	Δ 1-2	Δ 2-3	Δ 3-F	1-2 Avg	3-F Avg	Δ
Class 1	75.53	71.32	61.95	67.95	-5.57%	-13.14%	9.69%	73.42	64.95	-11.54%
Class 2	67.67	68.76	66.00	62.55	1.62%	-4.02%	-5.23%	68.21	64.27	-5.78%
Class 3	69.67	64.67	61.87	62.47	-7.18%	-4.33%	0.97%	67.17	62.17	-7.44%
Average	70.95	68.25	63.27	64.32	-3.71%	-7.16%	1.81%	69.60	63.80	-8.25%

MEDIAN

	Test 1	Test 2	Test 3	Final	Δ 1-2	Δ 2-3	Δ 3-F	1-2 Avg	3-F Avg	Δ
Class 1	78.00	77.00	65.00	75.00	-1.28%	-15.58%	15.38%	77.50	70.00	-9.68%
Class 2	67.00	73.00	64.00	66.00	8.96%	-12.33%	3.13%	70.00	65.00	-7.14%
Class 3	71.00	62.00	57.00	62.50	-12.68%	-8.06%	9.65%	66.50	59.75	-10.15%
Average	72.00	70.67	62.00	67.83	-1.67%	-11.99%	9.39%	71.33	64.92	-8.99%

The experimental group had significantly better improvement rates. The change in average scores from test 2 to test 3 (11.46% increase) is a substantial change from the test 1 to test 2 rate (2.42% decrease). Among the students who completed the entire course, there was still a small increase in performance on the final exam, a 2.71% increase. The glaring difference in the performance of the two groups can best be shown with the difference in performance on the first two exams versus the last two exams. For the experimental group, this came to a 14.18% increase in the average scores.





AVERAGE

	Test 1	Test 2	Test 3	Final	Δ 1-2	Δ 2-3	Δ 3-F	1-2 Avg	3-F Avg	Δ
Class 1	65.89	67.33	71.53	71.26	2.18%	6.23%	-0.37%	66.61	71.39	7.18%
Class 2	58.17	67.33	68.33	70.83	15.76%	1.49%	3.66%	62.75	69.58	10.89%
Class 3	58.93	50.07	66.27	64.87	-15.05%	32.36%	-2.11%	54.50	65.57	20.31%
Class 4	55.50	60.83	65.67	72.00	9.61%	7.95%	9.64%	58.17	68.83	18.34%
Average	59.62	61.39	67.95	69.74	3.13%	12.00%	2.71%	60.51	68.84	14.18%

MEDIAN

	Test 1	Test 2	Test 3	Final	Δ 1-2	Δ 2-3	Δ 3-F	1-2 Avg	3-F Avg	Δ
Class 1	66.00	58.00	72.00	70.50	-12.12%	24.14%	-2.08%	62.00	71.25	14.92%
Class 2	57.50	65.50	68.50	69.50	13.91%	4.58%	1.46%	61.50	69.00	12.20%
Class 3	59.00	49.00	69.00	71.50	-16.95%	40.82%	3.62%	54.00	70.25	30.09%
Class 4	58.50	59.00	66.50	74.50	0.85%	12.71%	12.03%	58.75	70.50	20.00%
Average	60.25	57.88	69.00	71.50	-3.58%	20.56%	3.76%	59.06	70.25	19.30%

Discussion

The experimental group's data supports the effectiveness of this active learning technique. The increase in average performance supports the idea that students were obtaining the knowledge on a much deeper level. The consistency of this increase through the final exam also supports the idea that student retention was also improved.

However, a few things should be taken into consideration. One fewer section was covered in the Fall 2012 semester compared to the Spring 2012 semester. The active learning techniques did take more class time than standard lecture. In addition, this process does not seem to be particularly viable for classes of more than 20 students. Individual attention must be given by the instructor to each individual group for a short period of time during group sessions. This becomes much more difficult with an increase in the number of students. Larger groups could be used (the author currently uses groups of size 4 in his Math 210 course), but the efficiency of the work done appears be inversely proportional to the group size (too much socializing, though perhaps this phenomena requires experimental verification). The time spent refocusing the groups that are getting off task could be better spent with other groups focusing on the task at hand.

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All for One and One for All: Algebra as Generalized Arithmetic

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Abstract:
This article compares algebra and arithmetic and stresses the importance of linking the two related topics. Through multiple examples the authors show not only how algebra and arithmetic are connected, but also why it is important to stress that connection.

Every teacher of high school and college level mathematics has seen the classic mistake $\frac{x+y}{x} = y$. But, rarely does a student "cancel the threes" in $\frac{3+4}{3}$ and get 4. So, where is the disconnect? How is it that a student can understand that the 3+4 is inherently in parentheses, and not see it for the $x + y$? It comes in with the presentation of algebra. Part of the problem is that algebra is not truly addressed in many curricula until long after basic arithmetic has been mastered. But, this cannot be the only problem. As far back as 1988, the National Council of Teachers of Mathematics (NCTM) recognized a need for algebra to be integrated throughout grades K-12 (*Curriculum*, 1988). Yet, we still have an alarming majority of students who talk happily about arithmetic and then shudder when reminded of the horrors of algebra class. In the United States, algebra is treated as its own topic aside from arithmetic. Sure, some arithmetic is involved, but algebra is algebra and arithmetic is arithmetic. Therein lies the problem. According to Cai and Moyer (2008), "If students and teachers routinely spent the first six years of elementary school simultaneously developing arithmetic and algebraic thinking…, arithmetic and algebra would come to be viewed as inextricably connected" (p.170). It is key to point out, that the authors are not asking teachers to create this connection, but to highlight the connection that is inherently there. More recently, in 2000, NCTM released *Principles and Standards for School Mathematics* which includes an algebra standard for all grade levels (NCTM, 2000).

The Common Core State Standards (CCSS) do support this approach to mathematics. In fact, one of the CCSS Mathematical Practices is as follows:

CCSS.Math.Practice.MP2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects (*Common Core*, 2012).

The CCSS have an "Operations and Algebraic Thinking" category for every grade level up until algebra is formally introduced in sixth grade. The phrase "by using…equations with a symbol for the unknown number to represent the problem" appears multiple times under this category for first, second, third, and fourth grade. By sixth grade CCSS has an entire section of standards under the heading "Apply and extend previous understandings of arithmetic to algebraic expressions" (CCSS, 2012).

The introduction of algebra at earlier levels of mathematical development is actually not a new concept. Chinese schools have been doing this for generations. According to a study by Cai and Hwang, Chinese students are far more likely to use algebraic strategies while U.S. students rely much more heavily on arithmetic. They attribute this to their mathematics curriculum which introduces algebra at a very young age (Cai & Hwang, 2002).

Many articles have been written in favor of viewing algebra as generalized arithmetic, and it appears that the CCSS want teachers to lean in this direction, but few examples are typically given. Some skeptics may wonder how one could incorporate algebraic ideas into younger grades. The authors have compiled some examples starting from the basics and ranging to more advanced mathematics. A few of these examples are commonly seen today, but many have hidden under the surface of mathematics for too long.

Example 1 Missing Addend and Missing Factor Models

Anyone who has taught subtraction and division is familiar with these models even if not by name. When a student is asked to evaluate $13 - 6$, they are taught to think "6 plus what is 13?" Many activities will even write 6 + □ = 13. The empty box in itself is a variable and the relationship between the two problems introduces students to the inverse nature of addition and subtraction. Similarly, the missing factor model for division proposes that $36 \div 9$ be thought of as "9 times what is 36?" These models are stated explicitly (though not by name) in CCSS 1.OA.B.4 and 3.OA.B.6 (CCSS, 2012). As these models are widely used today, we will not take the time to flesh them out here.

Example 2 Multiples and Multiplication Properties

When talking about multiples of numbers, students are told that the multiples of three are 3 times *some number* (which at this point in their education "number" means whole number). This in itself is algebra. Merely replace *some number* with a blank space or an empty box and you have introduced a variable. A fortuitous side effect of this is that students are led to the idea of a function with inputs and outputs. As different numbers are inputted into the box, different multiples of three are produced.

Jae Meen Baek (2008) has shown that young elementary students inherently understand the associative property of multiplication and the distributive property. Before being introduced to the standard algorithm when asked to multiply two and three digit numbers, students will naturally break the numbers apart and multiply the pieces. The students intuitively think algebraically in order to make the multiplication easier. In Baek's example, all the teacher had to do was formalize their thoughts and the students were able to easily solve for *b* using the distributive property in the following: $(4 \times 8) + (2 \times 8) = b \times 8$. Many college algebra students would approach this equation by simplifying what is in the parentheses by multiplying, adding the two products, and then dividing both sides of the equation by 8. It does not occur to them that four groups of eight and two groups of eight make six groups of eight. (Did you catch it?) By including the algebra back when they first learn the property, students are more equipped in the future to recognize the arithmetic in the problem, and not see it as a random string of steps.

Another positive consequence of introducing abstract algebraic thinking with the distributive property is that this idea can lead to deductive reasoning. Why is an even number plus an even number always an even number? Because each even number is $2 \times a\ whole\ number$, so "2 times *a whole number* plus 2 times *another whole number* equals 2(*a whole number* + *another whole number*)" or in other words, $2m + 2n = 2(m + n)$.

Example 3 Arithmetic Sequences and Patterns

Students are exposed to patterns from the very beginning of elementary school, but the patterns are presented purely arithmetically. From the authors' experience, freshmen college students given an arithmetic sequence, such as 4, 7, 10, 13, …, are able to immediately recognize the pattern. When asked to find the next term or the tenth term, they all quickly get to work and come to the answer. When asked to find the 100^{th} term, they either give up or fill an entire page with scratch work. Here is a perfect opening for algebraic thinking. Typically the dialogue goes something like this

Teacher – How did you get your terms before?

Students – I added three.

Teacher – If you start at the four, how many threes did you need to add to get to the *second* term?

Students – One.

Teacher – If you start at the four, how many threes did you need to add to get to the *third* term?

Students – Two.

Teacher – If you start at the four, how many threes did you need to add to get to the *fourth* term?

Students – Three.

Some students typically see the pattern at this point. Then with discussion they can be led to conclude that the explicit formula for the *n*th term is $4 + 3(n - 1)$ because you start at 4 and add a 3 for every term except the first one. The most advanced knowledge required to see this logic is that multiplication is repeated addition. In fact, if we had started at 0, the sequence would have just run through the multiples of 3.

Arithmetic sequences can also be examined through geometric patterns. Consider the following sequence of shapes.

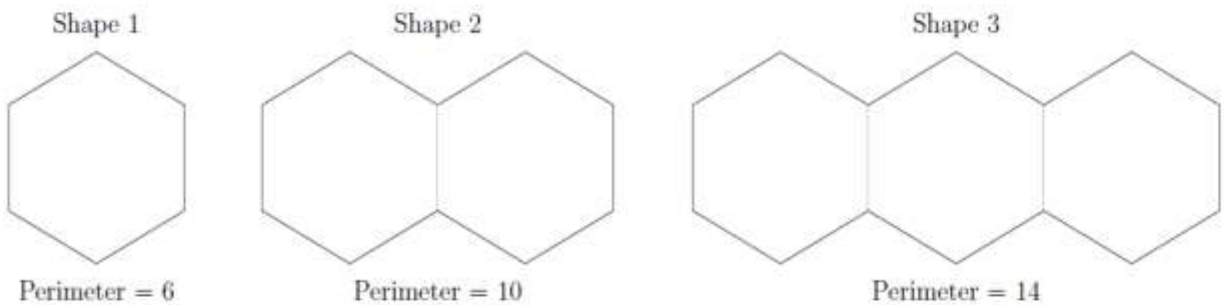

Figure 1

If this pattern is continued indefinitely, with each progressive step the perimeter increases by 4. Upon examination of the pictures, the reader will notice that every hexagon contributes 4 edges to the perimeter except that the first and last hexagons have one extra edge contributing. Thus the perimeter is 4 times the number of hexagons plus the two extra edges (i.e. $4k + 2$). Again, the logic required in creating the formula is nothing beyond knowing what a hexagon is and counting sides. It is pure quantitative reasoning. Many elementary level curricula deal with this idea using manipulatives such as straws or toothpicks, but they do not always stress the "logic to algebraic formula" connection. By high school, the CCSS expect students to "Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms" (CCSS, 2012, HSF-BF.A.2). But, the basis for this begins back in elementary school whenever students look at patterns.

Example 4 <u>Geometric Series</u>

Many calculus students struggle with the concept of infinite series when, in reality, the fundamental ideas behind infinite series are purely arithmetic in nature. Consider the following representation of a rational number with a finitely terminating decimal expansion in [0,1]. The number 0.12345 can be written as the sum

$$\frac{1}{10}+\frac{2}{10^2}+\frac{3}{10^3}+\frac{4}{10^4}+\frac{5}{10^5}$$

This sum is merely an algebraic expression for the base 10 representation where the power of 10 in the denominator determines the place-value of each digit and each term in the numerator denotes the particular digit.

This can be more easily conceptualized if it is noted that this sum really has the form

0.1 + 0.02 + 0.003 + 0.0004 + 0.00005.

More generally, any finitely terminating decimal expansion of the form $x = 0.a_1a_2a_3\ldots a_n$ may be written as

$$\sum_{j=1}^{n}\frac{a_j}{10^j}=\frac{a_1}{10}+\frac{a_2}{10^2}+\ldots+\frac{a_n}{10^n},$$

where n denotes the position/place-value and each $a_j \epsilon \{0,1,2,3,\ldots,9\}$ represents the corresponding digit.

This type of arithmetic-to-algebraic method can be valuable when students progress in their mathematical courses and begin to learn about non-terminating, repeating, or irrational expressions. Indeed, many irrational numbers of great importance can be written as an infinite series as well as rational numbers whose decimal expansions repeat. Moreover, this is also useful when learning about other bases (for example, a base three expansion will have digits 0, 1, or 2 with place value determined by a base of 3 as opposed to a base of 10). From personal experience, part of the problem students have with series in second semester calculus relates to the fact that they have not had enough exposure to the algebraic ideas behind infinite series and something as simple as this can lay groundwork at an early level.

One CCSS for high school algebra states, "Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments*" (CCSS, 2012, HAS-SSE.B.4). By using this idea combined with place-value for decimals, students can be transitioned into generic infinite series.

Example 5 Fractions

Consider the classic mistake from the introduction: $\frac{x+y}{x} = y$. There are so many ways for students to recognize their mistake, if they view the fraction through the lens of arithmetic. They are ignoring order of operations. They are treating addition and division as inverses. They are somehow getting that the *x*'s cancel to both the multiplicative identity, one (the denominator's *x*) and the additive identity, zero (the numerator's *x*) at the same time.

In Chapter 12 of *Reconceptualizing Mathematics*, Nickerson, Sowder, and Sowder (2010) provide beautiful description of working fractions side-by-side: one with numbers only, and a similar one with variables. This process allows students to notice their arithmetic steps with the numbers and mimic those steps with the variables.

Consider the addition and subtraction fractions. Students can arrange a problem with numbers and a problem with variables side-by side as described above, pay attention to how they find a common denominator with the numbers, and repeat the process with variables as shown in Figure 2.

$\frac{3}{40}$	$+$	$\frac{2}{70}$	Need LCD	$\frac{3}{xy}$	$+$	$\frac{2}{yz}$
$\frac{21}{280}$	$+$	$\frac{8}{280}$	Multiply by "missing" piece for LCD	$\frac{3z}{xyz}$	$+$	$\frac{2x}{xyz}$
		$\frac{29}{280}$	Combine the numerator	$\frac{3z+2x}{xyz}$		

Figure 2

One of the authors recently used this section from *Reconceptualizing Mathematics* in a mathematics course for education majors. Something of interest this author noticed was that this process actually exposes holes in a student's arithmetic. One group of students all equivocally agreed that the denominators 40 and 70 have LCD 280, but could not see how to find the LCD when the denominators were *xy* and *yz*. When asked how they arrived at 280, none of them could answer. No wonder they cannot find a common denominator if they cannot remember the basics of finding the LCM of two numbers! This is a bi-product of the pattern-heavy/ algebra-light focus of "before-algebra" mathematics. When the idea of least common multiple is introduced in sixth grade, along with viewing the patterns created, students need to see the abstract idea of what is going on. Terms are broken down into their basic parts, primes in the case of whole numbers and primes and variables in the case of algebraic terms. Then a product must be created from those basic parts so that each original number/term is there inside the product.

Notice how in the final step of Figure 2 the first numerator combines to a monomial and the second numerator must remain a binomial. This same group of students wanted to collapse the second numerator to one term, 5*xz*, to mimic their action on the first fraction. Again, this brought to the surface a hole in their understanding of arithmetic. They had forgotten the inherent meaning of addition and what it means to combine like terms. According to Nickerson et al. (2010), this is a very common mistake with algebra. Students think that they understand something (like $21 + 8 = 29$), but they do not understand it in depth enough to abstract it to variables.

Through this one problem, two important mathematical ideas that the students were missing (or had forgotten) were exposed and the teacher was able to go back and fill in the gaps. If the algebraic side of things had been introduced much earlier, these issues would have come to light sooner, and the students would not have had to struggle as much in math.

Example 6 <u>Exponents</u>

Current teaching methods do not do justice to exponents linking the arithmetic to the algebraic. In algebra, students often spend so much time learning mindless algorithms and "rules" that they hardly have a conceptual idea of what exponentiation is all about. Simply put, exponentiation to a positive integral power is repeated multiplication; exponentiation to a negative integral power is repeated multiplication of the multiplicative inverse (we will deal with the issue of inverses in example 8). In arithmetic, we teach the idea of reduction of rational numbers (which is essentially an algebraic process) so why not draw the connection to exponentiation along with it? A fraction simplifies due to the presence of a common factor (i.e. the numerator and denominator are not relatively prime) which is the same reason that $\frac{2}{8} = \frac{1}{4}$ or algebraically speaking

$$\frac{x^n}{x^m} = x^{n-m},$$

where *n* and *m* are integers. However, many calculus students have a difficult time seeing simple things such as

$$\frac{x^n}{x^{n+1}} = \frac{1}{x}$$

where $x \neq 0$. It is almost as if they freeze up when they see exponential notation but therein lies the problem; they aren't struggling with the notation. (This is historically how people have decided to denote exponentiation). Rather, they struggle with the ideas behind it. But, in reality, the basic rules of rational exponents are really basic arithmetic properties.

When moving beyond rational exponents, things become more difficult. It is not reasonable to expect students to grasp the meaning of a number like $e^{\sqrt{2}}$ simply by being given a simple explanation that exponentiation to integral powers is repeated multiplication. How does one multiply *e* "square root of 2" many times? It is also detrimental to offer silly explanations such as "plug it into your calculator" to the question "what is $e^{\sqrt{2}}$?" Such a response gives no insight into what this number actually is and further emphasizes mindless algorithmic repetition. Simply put, general exponential functions like e^x are not algebraic objects as functions on the real line but we treat them as if they are. The closest a non-calculus level explanation can get is to graph the function for rational exponents (which are understood) and connect the values obtained in such a way as to create a "smooth" curve. But in reality, this is no easy task. It is no wonder that calculus students really have little understanding of such things; we treat exponential functions as if they are trivial when students are barely given the tools to understand the basic algebraic principles that lead to the generalization of exponentiation of irrational numbers in calculus.

The CCSS mention exponential functions regularly throughout the upper grade levels. It is no surprise that all specific examples given in these standards use rational exponents, as these are understandable objects. Teachers just need to be aware that when moving beyond rational exponents these functions cannot be seen as simple extensions. (And when introducing logarithms as inverses of exponential functions, these same issues are present.)

Example 7 Polynomials and Place-value

Another example given in *Reconceptualizing Mathematics* involves operations on polynomials. A polynomial with positive integral coefficients can be viewed as a number in expanded form. Each power of *x* just relates to the place-value in the number. For example, $(3x^2 + 2x + 1) + (5x^2 + x + 4)$

should be treated like $321 + 514$ and like terms/place-values should be added to like terms/place-values. In fact, the polynomials are a little easier, because you do not need to carry over to the next term if your coefficient is bigger than 9. Similarly, when multiplying two polynomials you are merely multiplying each term by each term just as you would multiply each place-value by each place-value in the number. Again, with the added bonus that you do not have to worry about carrying over if your coefficient is too big. The distributive property on the product of polynomials (which is often referred to by that dreaded acronym the authors refuse to use) is precisely the partial products algorithm of multiplication. For example, $(43 \times 21) = (40 + 3)(20 + 1) = 800 + 40 + 60 + 3 = 906$ follows the same steps as $(4x + 3)(2x + 1) = 8x^2 + 4x + 6x + 3 = 8x^2 + 10x + 3$.

This similarity between digits with their place-values and coefficients with their power of *x* is even more obvious when one looks at the manipulatives involved for each. Figure 3 shows the major manipulatives for representing place-value (base ten blocks) and polynomials (algebra tiles).

111 in base ten blocks $x^2 + x + 1$ in algebra tiles

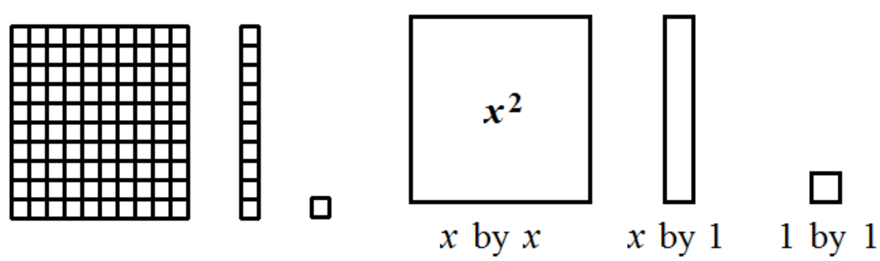

Figure 3

The only difference between these tools is that with algebra tiles you do not know how many units make a long (x) nor do you know how many longs make a flat (x^2).

Example 8 “Canceling”

One aspect of arithmetic that is taught early on is inversion. That is, the number 2 has a multiplicative inverse of $\frac{1}{2}$. However, this is rarely taught in an algebraic context. That is, what is it that fundamentally makes something a multiplicative inverse? The idea of an identity is easily understood; the number 1 is the multiplicative identity as $a \times 1 = 1 \times a = a$ for any real number *a*. Thus, something has a multiplicative inverse, which we *denote* simply by a^{-1} provided that $a^{-1} \times a = 1$. In other words, when the *operation in question* is multiplication, we have $a^{-1} = \frac{1}{a}$. When the operation in question is addition, we choose to use the notation $-a$, but the idea is the same. The additive inverse of *a* is whatever number when added to *a* will achieve the additive identity (zero).

So, why is it beneficial to teach inversion in an algebraic context? Firstly, it clearly elucidates why 0 cannot have a multiplicative inverse (and why 0 has an additive inverse). For if it did, there would be a number *b* for which $b \times 0 = 1$ but this implies that 1 = 0. (Incidentally, as a side note, when systems of linear equations are taught in algebra, students typically do not have enough of a logical foundation to understand that we begin by *assuming* that a solution exists. Though, CCSS HSA-REI.A.1 stresses exactly this point (CCSS, 2012).)

Secondly, this might help clear up issues that calculus and trigonometry students have when they see the notation $\cos^{-1} x$. The additive inverse has its own notation, but no other inverse does. The rest all rely on the -1 exponent. When considering $\cos^{-1} x$, inversion, in this context, is inversion of functions with respect to composition, not multiplicative inverses of the real numbers. A good many students simply think that inverse cosine is its reciprocal, $\sec x$. The CCSS deal with inverse functions in HSF-BF.B.4a-4d, but none of these standards explicitly mention the important fact that these are inverses with respect to composition.(CCSS, 2012) In light of this, it becomes awkward for students that we use the notation f^{-1} in algebra to denote the inverse of a function *f* without hardly any explanation as to why this isn't the multiplicative inverse 1/*f*. It can't be mere coincidence that students, who are constantly told that $a^{-1} = \frac{1}{a}$ with little to no explanation as to why, become confused when, in algebra, we tell them that this is not true for f^{-1}. Ultimately, this goes back to the fact that we separate the idea of inversion in arithmetic and the idea of inversion in algebra.

Conclusion

Obviously, some of these techniques are more aptly applied to younger grade levels and some of them are more aptly used when algebra is being more formalized, but the point is that algebra *is* arithmetic. Anyone who truly understands arithmetic can understand algebra and moreover, by using algebra, gaps in one's knowledge of arithmetic can be discovered. Mathematics has long been erroneously viewed as a linear progression of ideas. You learn one thing and move on to the next. But really, math is a beautifully woven web where everything inter-relates. Algebra and arithmetic are intrinsically linked and should not be separated.

If we as teachers do not adjust the way we present algebra (and abstract reasoning in general) we will continue to create people like Roger C. Schank (2012) who believe "The average person never does abstract reasoning. If abstract reasoning was so important, we could teach courses in that;" i.e. people who are so disconnected from what abstract reasoning is that they do not even recognize that they had to use abstract reasoning to come to their conclusion against it!

The authors welcome any comments, especially more examples of how arithmetic and algebra are linked. Please feel free to email Heather Gamel at heather.gamel@nicholls.edu.

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Math Apps for Students of All Ages

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Abstract:
Apple's iPad, introduced in 2010, is a newcomer to the world of education. Part of the attractiveness of the iPad is the ability to download apps to the device. These apps allow a person to "customize" the iPad for their own use. Students can download various apps to practice math skills, to watch videos on solving problems, and to learn basic math concepts. The paper will discuss 12 math-related apps for students of various ages.

INTRODUCTION

In April 2010, a great change was made to the world of technology with the introduction of Apple's iPad. Since then, over 100 million iPad have been sold (AP, 2012). Initially designed as a device for media consumption, apps available for download can turn an iPad into a variety of tools/devices. Using a calculator app, one can compute a mortgage payment or convert a temperature. Certain productivity apps allow for managing a bank account while other productivity apps allow for the creation of documents, spreadsheets, and presentations. Some apps function as a GPS device while others function as weather radar. As one can see, an iPad is more than a media consumption device. With the number and ability of the different available apps, the use of an iPad can have an infinite number of possibilities.

iPads in Education

Apple has sold two times as many iPads as Macs to K-12 schools and colleges (Ogg, 2012). Apple's CEO Tim Cook has stated, "The adoption of the iPad in education is something I've never seen in any technology" (Ogg, 2012, para. 3). Many schools are issuing iPads to students as a textbook replacement (Reitz, 2011). Ireland and Woollerton (2010) believe "the iPhone has had and will continue to have an impact on learning" (p. 35). They also believe the iPad "will truly change and revolutionize" teaching in the future (p. 35).

But why is the iPad so appealing to education? Ireland and Woollerton (2010) cite the size of the iPad and its screen as reasons supporting their opinions on the use of iPads for learning. Quillen (2011) agrees with the appeal of the physical aspects of the iPad including the battery life of approximately 10 hours and a weight of a little more than a pound. Besides being small and very portable, the devices are relatively durable (Shah, 2011; Marmarelli & Ringle, 2011). With an LED backlit display with lighting that adjusts to the environment (Valstad, 2010) and a screen resolution of 2048 x 1536 (Apple, 2013), the quality of graphics on iPads is astonishing (Johnston & Stoll, 2011). Due to Apple's operating system, users have "virtually no threat from malware of any kind" (Brovey, 2012, para. 7).

Besides being used as textbook replacements, students are using iPads to access downloaded apps. At Apple's 2012 Education Event, Senior Vice President of Marketing Phil Schiller announced that over 20,000 education and learning apps have been designed for the iPad (Rao, 2012). Apple controls the quality and number of apps available in its App Store, so users can be assured that the apps will perform as intended (Johnston & Stoll, 2011).

iPads in the Classroom

But how can these apps be used in the classroom? Berk (2010) encourages teachers to use technology to supplement their lessons. In a research project, Bradley (2012) found that students were so engaged in the use of their iPads in classroom, they would often do more work than required. In addition, Bradley (2012) reported that (a) students were having more control of study activities, (b) teachers were working more as facilitators, (c) teachers were having more time to work with students individually, and (d) classrooms were becoming more student-centered.

Valstad (2010) emphasizes that choosing a suitable app is essential for using iPads in the classroom. Johnston and Stoll (2011) suggest using the iPad to change how students are taught mathematics; the use of an iPad can change how students view images and "manipulate the magnificent images on the screen, to pinch and zoom, to spin and flip, to tap and slide" (para. 22).

The Houghton Mifflin Harcourt study on the effects of the HMH Fuse app on student achievement reported that 78% of the students using the HMH Fuse app scored “proficient” or “advanced” on the California Standards Test while only 59% of their counterpoints using the textbook version scored within the same range (Barseghian, 2012).

Because of the popularity of the iPad, the quantity and quality of apps, and how students are embracing the use of iPads to facilitate their studies, the author decided to review several math-related apps. The apps reviewed range from entry-level math concepts through high school. In addition, a couple of very good calculator apps are included in the review.

REVIEW OF THE APPS

King of Math is inexpensive app ($0.99) that is designed for iPhone and iPad. Due to the topics covered, this app would appeal to elementary school age students through middle-school age. Covered in the app are different concepts including addition, subtraction, multiplication, division, arithmetic, geometry, fractions, powers, statistics, and equations. Students can improve their math skills by answering math questions and improving their total score. The app is very popular with over 3 million downloads and has a rating of 4.5 out of 5 stars. The website for King of Math is http://www.oddrobo.com.

My Math Flash Cards is a free app that can be used to master basic elementary math facts. The app is designed for both iPhone and iPad. The student can select which math concept to practice, including addition, subtraction, multiplication, or division. There are settings for the number of digits (1-9 or mix) and the number of questions in each set (5-50, by 5s). In addition, there are settings for whether or not to have a timer, help, and sound. The app has a rating of 4 of 5 stars. For more information about this app, visit http://powermath.wordpress.com/.

Math Evolve: A Fun Math Game is designed for both iPhone and iPad. This educational game is inexpensive ($1.99) and has received excellent ratings. This app is designed for ages 6 and up but can also be fun and challenging for adults. Math Evolve combines math practice with an arcade-style game. The game has two modes; in the "story mode", the players embark on a math adventure across different environments while the "practice mode" allows players to learn math facts. The app has a rating of 4.5 out of 5 stars. More information and a preview video about the game can be found at http://mathevolve.com.

Flexigons is an inexpensive ($0.99) app designed for the iPad. In Flexigons, players can move puzzle pieces using geometric transformations of translation, reflection, dilation, and rotation. Using this app, players can apply geometric principles. The game has 45 different levels in 3 different modes. This app was developed with the support of the Bill and Melinda Gates Foundation. The current version of the app has a rating of 5 out of 5 stars. The website for this app is http://flexigonsapp.com/.

DragonBox + Algebra is the most expensive app ($5.99) reviewed. This app is designed for both iPhone and iPad. Designed for ages 6 through 16, this app presents a gradual learning of algebraic elements. The player learns at his or hers own pace. There are ten progressive chapters (five for learning and five for training) and 200 puzzles. The birth and growth of a new dragon signifies the player progressing and completing a new chapter. The app rates 5 out of 5 stars. The website for this app is http://dragonboxapp.com/.

Doodle Numbers Quiz is an addictive free app that is available for both iPhone and iPad. This app requires the player to solve a series of simple pattern-matching puzzles; there are 64 puzzles available to solve. Instead of matching colored balls and shapes, the player is matching numbers using basic math. To remove puzzle pieces, the player must match neighboring pieces of equal value or select neighboring pieces that add to 10. The app has excellent reviews with a rating of 4.5 out of 5 stars. For more information about the app, visit http://redspell.ru/games/for-ios/doodle-numbers/lang-en/.

Sakura Quick Math is an inexpensive app ($1.99) available for iPhone and iPad. This app improves math skills by having players solve problems while racing the clock. Using advanced handwriting recognition, the player "writes" the answer on the screen. This app is described as being perfect for improving mathematics ability for students in grades 3 through 6. The app rates 4 out of 5 stars. The website for the app is located at http://www.getshinythings.com/quick-math.html.

Calculator is a very inexpensive app ($0.99) available for iPad. This app contains 12 different calculators including Scientific, Currency Converter, Unit Converter, Date - Time, Constants, Tip calculator, Biorhythm calculator, Equation Solver, Statistics, Base conversion, Graph, and Finance. There are three calculating modes: Standard, String, and RPN and the precision is to 31 digits. The app has a rating of 4.5 out of 5 stars. For more information about this app, visit http://www.ppclink.com/apps/index.php?page=hicalc.

Quick Graph: Your Scientific Graphing Calculator is a free app. This app is available for iPhone and iPad. The app will display graphs in both 2D and 3D for up to six equations simultaneously. Specific points of an equation can be evaluated using the evaluate feature. Commonly used equations can be stored in a library. Graphs can be emailed, saved to photo library, or copied to the clipboard. The current version of this app rates 5 out of 5 stars. For more information about this app, visit the website at http://kzlabs.me/quick-graph/.

Mathemagics – Mental Math Tricks is an inexpensive app ($0.99) that is available for iPhone and iPad. Using this app, students can learn and practice tricks of mental math calculation. There are more than 50 tricks to learn and three modes of play to practice the tricks through various levels of proficiency. In the Practice mode, the students can practice the tricks over and over. If additional help is needed, the students can view the lesson for the trick again. In the Play mode, the students answer random tricks in random order. The use of this app is an excellent preparation for the math sections on standardized test. This app rates 4.5 out of 5 stars. The website for this app is located at http://www.bluelightninglabs.com/ .

AP Exam Prep is a free app available for both iPhone and iPad. This app contains 25 questions from each of the 15 different AP topics. With each topic, an additional 450 to 500 questions with answers are available. Additional questions for both AP Calculus and AP Statistics are $8.99 each. Problem areas can be tagged and quizzes can be created from these flagged questions. The current version of the app rates 5 out of 5 stars. The website associated with this app is located at http://www.gwhizmobile.com/gWhiz/Apps.php .

Video Calculus is a free app designed for the iPad. Included in the free app is access to 15-featured lessons resulting in over 2 hours of videos. Additional lessons may be purchased through the app. The lectures are 8-20 minutes in length and can be bookmarked. In addition, notes can be typed and saved for each lecture. Because the videos are not permanently stored on the iPad, a Wi-Fi connection is required to view the videos. The video lectures are presented by award-winning professor, Edward Burger. This app rates 4.5 out of 5 stars. For more information about this app and video, visit the website http://www.thinkwell.com/student/product/calculusab .

CONCLUDING THOUGHTS

Combining the hardware and the variety of apps available for download, today's iPad is not just a media consumption device. From what the author has experienced, read, and heard anecdotally, the iPad is an excellent tool that can be used to practice math skills, to watch videos on solving problems, and to learn basic math concepts. Today's students like the immediate feedback that an iPad app can provide. Keeping in mind the different possibilities of how the iPad could be used to help students in math, the 12 apps reviewed cover a breadth of math topics and app abilities. There were literally thousands to choose from for the list. As the list was being compiled, newly found apps were added while others were removed. Due to the ever increasing number of educational apps, six months from now this list could be totally different.

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Guest Column

A Note from our Editor Emeritus – *Dr. Katherine Pedersen*

Colleagues and Friends:

Dr. Plaisance has graciously invited me to write a little note to the readers of the LATM Journal. It is a pleasure!!! Let me share some thoughts.

Sometime in the past, I researched the pattern of emphasis in mathematics teaching: i.e. basic facts vs. problem solving – to use very stereotypical terms. Roughly counting, the change from one emphasis to the other and, maybe, back again appeared to occur about every 30 years. I found something about facts vs. problem solving around 1930; I myself experienced a change from basic facts to problem-solving around 1960. Then, of course, about 1990-1991, a change in the teaching of mathematics was heavily funded and publicized. I am obviously looking for the change in the next 5-6 years. How is this relevant to us now and to the LATM Journal?

Practicing mathematics educators, those in and out of the classrooms, have been the one constant through all of these "pronouncements" of change. It is the classroom teacher-- or lecture-room teacher-- who lets us know what happens in the process of implementing a suggested strategy in the classroom. How does an educational theory get any verification: only through a teacher trying it and gauging the students' degree of learning. Without the practicing mathematics educator – and his or her sharing of classroom experiences-- ideas do not become actions.

I think that is our "job": to translate ideas into actions that help the students learn. All of us have heard of the "Sieve of Eratosthenes".. take a colander, throw all the positive integers into it, and let the prime numbers fall through the holes. I view practicing mathematics educators as my "Sieve of What Works".. throw all educational theories into this Sieve, and the practicing mathematics educators will help decide what works. The practicing mathematics educator will journal his or her experiences and share them with colleagues. This is where I see the LATM Journal. This last step, the sharing of ideas and experiences, is our "Measure" that the ideas which work will be continued and supported. We are the "Sieve" that separates ideas from working practices. And, the LATM Journal is our means of sharing.

With the greatest regard for LATM and the LATM Journal.

Katherine Pedersen, Ph. D.
Editor Emeritus, LATM Journal

Preservice Point of View: "Aha" Moments and Wishes

Nell McAnelly and DesLey Plaisance

This section of the *LATM Journal* is designed to link teachers and future teachers. Usually, in each journal, responses to a mathematical task by preservice teachers are presented. It is anticipated that these responses will provide insight into understanding, reveal possible misconceptions, and suggest implications for improved instruction. In addition, it is expected that this section will initiate a dialogue on concept development that will better prepare future teachers and reinforce the practices of current teachers. In this issue, a different approach was employed in order to initiate dialogue and "food for thought" for inservice teachers.

Approximately 100 preservice elementary education students enrolled in an upper level mathematics content course were surveyed. These students were in the third course of a sequence of three mathematics courses designed specifically for elementary education majors. The students were asked two questions:

1. What has been the biggest "aha" moment for understanding mathematics in the math classes that you have taken for teachers? Explain briefly.
2. What successful teaching strategy for math have you seen that you wish your K-12 teachers would have used more often? Explain briefly.

Think about these two questions. What do you think were the most common responses? Think back to your college mathematics courses. What were your "aha" moments? What strategies do you wish your K-12 mathematics teachers would have used? Are you using those strategies in your teaching of mathematics?

All of the responses to Questions 1 and 2 were read and categorized. One thing to note is that sometimes preservice teachers do not distinguish between mathematics content courses and mathematics methods courses. The intent of the first question was to find out what was happening in "math" class, but some responded with moments from their mathematics methods classes. In that it is still an “aha” moment in a class learning math, those responses were included in the summary of responses. We felt that the “aha” moment and the successful teaching strategy would provide that “food for thought” for mathematics teachers at any level.

Aha Moments

There were three main types of responses that kept appearing in the responses to Question 1. The response that was most often given as an “aha” moment was that there are multiple ways of solving problems. Some of these responses are as follows:

1) I have always told myself that I am not good at math because I didn’t understand the procedural way my school teachers taught it to me. Finding other ways to solve problems or to look at them has built my confidence for not only my math skills but teaching future students math.
2) Everyone learns math in different ways and there is no one single right way to do a math problem. Some may learn better with pictures whiles others may not like that and pictures may make them more confused. We, as teachers, should be open to different interpretations and ways students solve a problem.
3) The biggest “aha” moment was learning that it is important to show different ways to solve problems and to allow students to work the problems in different ways. This makes math less intimidating and allows students to develop at their own pace. This adds to a relaxing math environment, which is incredibly important.

The second most given response for Question 1 was that there were “aha” moments when teachers explained the meaning of the mathematics concepts and/or allowed the students to discover the meaning as opposed to memorization. Most of these responses referenced learning formulas and never understanding them. However, when the “meaning” behind the formula was discovered and/or explained, understanding was present for the use of the different formulas

The third most given response for Question 1 was that an “aha” moment occurred when students were assigned problems that allowed them to experience how their own students would feel when working certain types of problems. The majority of the responses described a specific example with place value. Students were given assignments with bases other than 10. Allowing students to work problems outside of base 10 provided a reference for elementary students learning base 10. Adult students cannot recall a time when they did not work base 10 problems without “carrying” and “borrowing.” This is done without thinking by that stage of life. But, looking at “carrying” a group of three in base three makes a student stop and think about how difficult it can be for an elementary student to understand “carrying” a group of 10 in base 10.

Wishes

Question 2 asked the future teachers to identify a successful teaching strategy that they wished their K-12 teachers would have used. The answer given by nearly half of the students surveyed was that they wished their teachers would have used manipulatives and more hands-on activities. Some of the responses are as follows:

1) I have seen a teaching strategy of using more hands-on examples and manipulatives. I wish my K-12 teachers would have used these more often when teaching math. It really benefitted my understanding and was much more successful and engaging than doing practice problems over and over and taking notes.
2) I wish my K-12 teachers would have used more manipulatives. Right now our mentor teacher uses some manipulatives and it definitely helps the students understand concepts better. I wish I would see it more in classrooms but I don’t because it usually takes more time and preparation.
3) I love using manipulatives when teaching any subject, especially math. I would have liked for my K-12 teachers to have used more manipulatives to help me, as well as other visual learners, better grasp mathematics topics.

The second most given response to Question 2 was that students wished their K-12 teachers would have explained why the mathematics works as opposed to memorization of steps for solving different types of problems. Students referenced the explanation of formulas just as they did in Question 1. Understanding the “why” and “how” allows them to utilize the mathematics more effectively without memorizing meaningless steps.

Two responses were given the same number of times for the third most given response. Students stated that they wished their K-12 teachers would have using drawings and more visual images in mathematics classes to allow for better understanding. They also stated that they wished their teachers would have recognized and allowed multiple strategies for working problems. Some students stated that teachers showed one method and that was the required method for homework and testing purposes. This response matched the most given response to Question 1.

It can be beneficial for inservice teachers to "listen" to what the preservice teachers are saying. Are there moments and strategies that you have forgotten about? Think about them and bring them back into your mathematics classroom!

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Mrs. Nell McAnelly *is Co-Director of the Gordon A. Cain Center for Scientific, Mathematical, Engineering, and Technology Literacy and is a mathematics instructor at LSU-BR. Her most recent teaching focuses on mathematics content courses for elementary education majors. McAnelly oversees numerous professional development projects for mathematics and science teachers from kindergarten through twelfth grade and is the Executive Director of the Quality Science and Math Grant Program. In addition, she is the Project Director for the NYSED/Common Core Eureka Math Curriculum Development Project. She presently serves on the LATM Executive Council as NCTM Representative.*

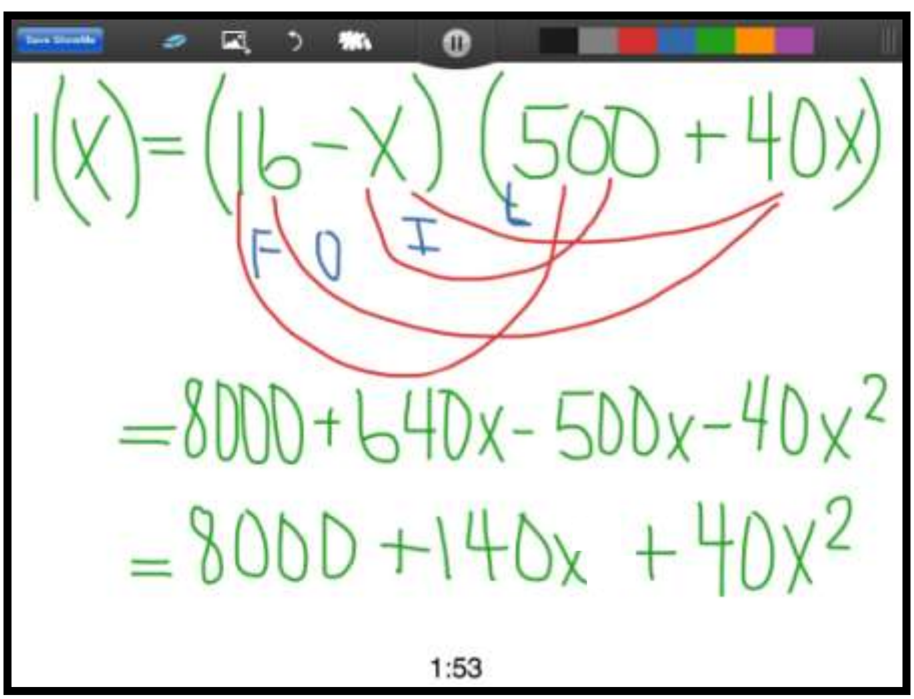
Tech Talk: Save a Tree, Use an App

Lori C. Soule

With more and more people joining the “going green” movement, I started thinking of ways to save paper while still helping students learn how to work math problems. I decided to take a look at a couple of iPad apps that could be used to demonstrate the steps in solving an equation. The apps had to be free, had to allow for recording the written steps to solving the equation, and had to allow for voice-over input.

The first app evaluated was ShowMe Interactive Whiteboard. The app met the three stated criteria. The use of the app is very intuitive. The red record button in the top middle of the screen has an obvious purpose. There are seven color choices for writing/drawing. Images from your photo library, built-in camera, or web search can be loaded onto the whiteboard. Recordings can be paused. Once the recording is complete, you can upload the recording and share it with friends. There is no time limit on the recordings and there is no limit on the number of recordings you create.

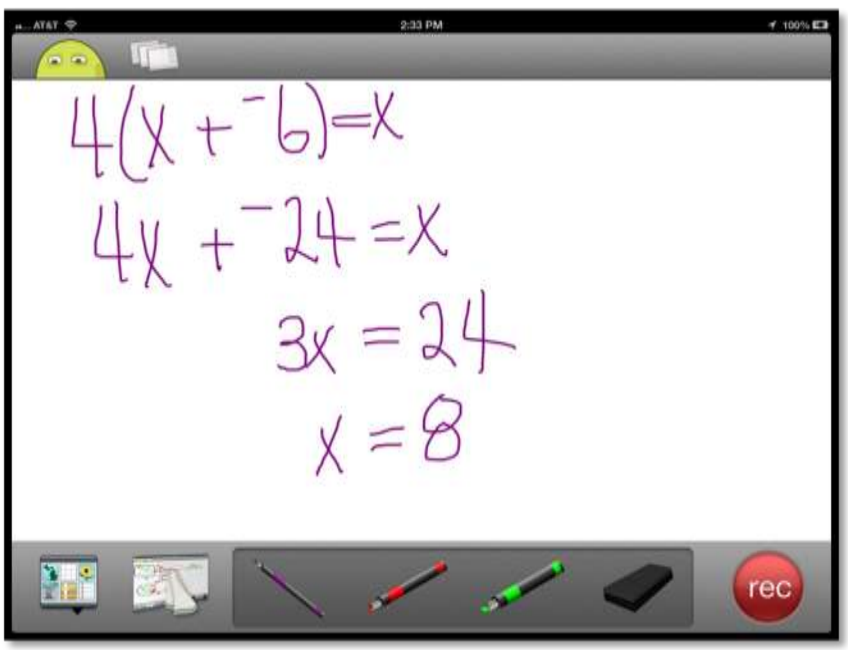
Figure 1. ShowMe Interactive Whiteboard



The website is http://www.showme.com/. By visiting the website, you have access to hundreds of ShowMe recordings. The recordings are grouped by subject area: math, science, language, English, social studies, and music. There are even Common Core lessons available on ShowMe. All of these recordings, including the ones you create, can be viewed on the web; an iPad is not a necessity for viewing.

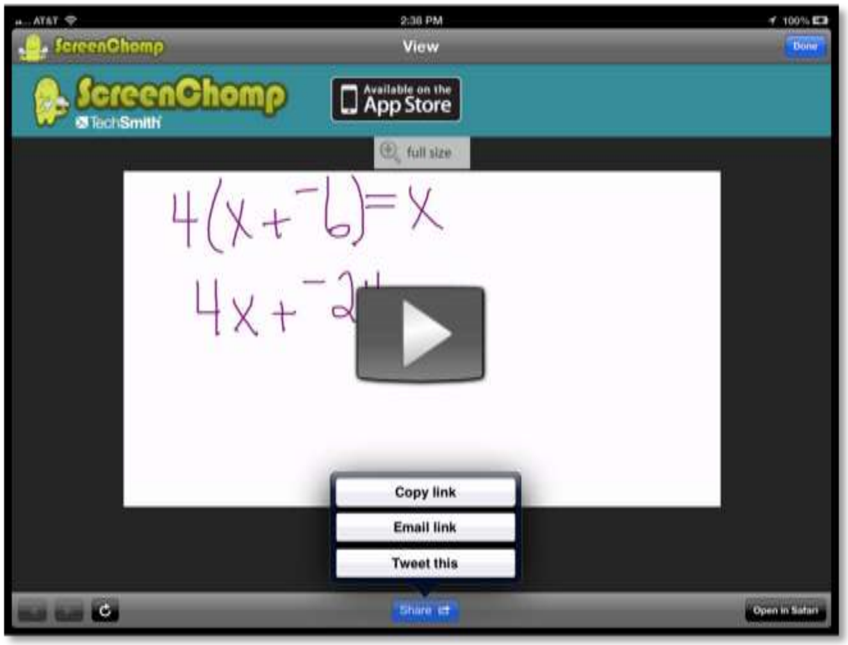
The second app evaluated was ScreenChomp. This app also met the three stated criteria. Like ShowMe, there is a large, red record button; it even contains the letters REC. There are 12 pen color choices and three pen widths. Holding your finger on a pen will bring up the color pallet and width choices. Images can be loaded as your background. These images can come from your photo library, built-in camera, or Dropbox account. Recordings can be paused and the screen can be "cleared" to give the effect of going to a new page.

Figure 2. ScreenChomp



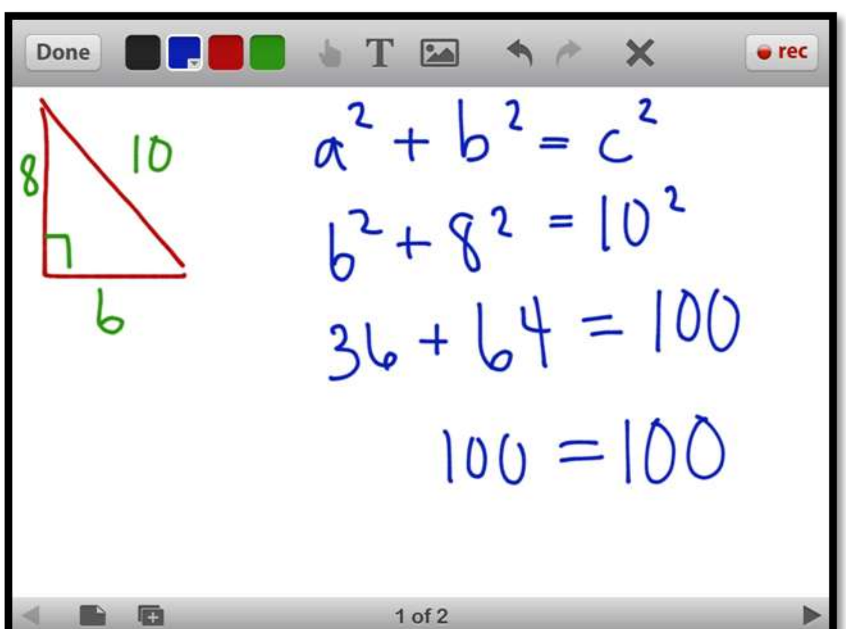
The link to the completed recording can be copied, emailed, tweeted, or opened in Safari. Videos can be downloaded as an MPEG-4 file. The website for ScreenChomp is http://www.techsmith.com/screenchomp.html.

Figure 3. Completed ScreenChomp recording



The final app evaluated was Educreations Interactive Whiteboard. Like the two other reviewed apps, this app met the three stated criteria. Like ScreenChomp, there is a red record button containing the letters REC. The ink color pallet contains 10 choices. Images can be loaded from your photo library, iPad camera, Dropbox account, or the Web. Recordings can be paused and resumed at any time. Different in this app is the ability to add text to any page. In addition, images can be animated by moving them around on the screen during a recording session.

Figure 4. Educreations Interactive Whiteboard



The website for this app is http://www.educreations.com/. The website contains hundreds of recordings from the subject areas of math, science, social studies, English, world languages, and the arts. Completed recordings can be kept private or shared with your students, your school, or the general public. Unlike the other two apps, you will need to create a free account with Educations.

As you can see, using any one of these three apps will allow you to demonstrate how to solve an equation without killing any trees. These apps are free and easy to use. Recordings can be shared in various ways. So in celebration of the growing "going green" movement, put away your paper and pen and download an app.

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